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MODELLING OF VISCOUS RESUSPENSION USING A ONE-FIELD DESCRIPTION OF MULTIPHASE FLOWS

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ABSTRACT

To investigate the behaviour of polydisperse suspensions in shear flow, a computational model is developed which includes the descriptions of hindered sedimentation and shear-induced particle migration. The flow of polydisperse suspension is modelled using a one-field description of multiphase flows, which is highly efficient and robust. Mechanisms affecting the transport of solid particles in shear flow are considered and included as closure correlations in the model. The model is applied to predict the distribution of solid particles in simple (homogeneous) shear and pipe flows.

NOMENCLATURE

a particle radius, m
$\mathbf{g}$ gravity vector, m/s^2
p pressure, Pa
$\mathbf{u}$ velocity, m/s
t time, s

ϕ solid volume fraction
$\dot{\gamma}$ shear rate, 1/s
ρ density, kg/m^3
μ dynamic viscosity, Pa.s
τ stress, Pa

Subscripts

f fluid
r relative
s solid

INTRODUCTION

Viscous resuspension is the entrainment of heavy solid particles into the lighter fluid carrier in the presence of a shear field. The shear induced flux of particles can be in a direction opposite to that of gravity. In many industrial applications and facilities, viscous resupension may occur in concurrence with settling and, therefore, has an important effect on the flow of suspensions and the operation of the equipment under consideration.

A number of experimental and analytical studies have been conducted to investigate the phenomenon of viscous suspension in pipe, channel and Couette geometries. However, little work has been done on computational modelling of viscous suspension, especially for polydisperse flows. The effects of turbulence have also been ignored in most experimental and analytical work devoted to viscous resuspension study.

In this work, a computational model is developed to investigate the behaviour of polydisperse suspensions in the presence of a shear field. The developed model is used to examine the viscous resuspension phenomenon in some typical flow conditions.

MODEL DESCRIPTION

One-Field Equations For Polydisperse Suspensions

It is assumed that a polydisperse suspension consists of one fluid field and N solid species. Following the approach of Rudman (1997) the solid-liquid mixture is considered as a continuum (fluid field) with effective properties dependent on the solid concentrations and the transport of the solid-liquid 'mixture' is described by one-field governing equations. The one-field equations for polydisperse suspensions are derived from the multi-field equation system describing the transport of each liquid/solid field:

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla(\mathbf{u} \cdot \mathbf{u} + C) = \mathbf{g} + \frac{\nabla \cdot \tau_f}{\rho}, \tag{1}$$

where $\mathbf{u}$ is the mixture velocity and τ_f is the bulk hydrodynamic stress per unit fluid volume. C is an extra convection term arising from the summation of the momentum equations of all fluid and solid fields (see Rudman (1997)).

The effective viscosity of the mixture is a function of the total solid concentration as follows:

$$\mu = \mu_f \left(1 - \frac{\phi}{\phi_{\max}}\right)^{-n} \tag{2}$$

where μ_f is the viscosity of the carrier fluid and ϕ_{max} is the maximum packing volume fraction.

The solid volume fractions are governed by particle transport equations which take into consideration both convection and diffusion as follows:

$$\frac{\partial \phi_i}{\partial t} = -\nabla \cdot (\mathbf{u}_{si}\phi) + \nabla \mathbf{J}_i, \quad i = 1..N \tag{3}$$

where $\mathbf{J}$ is diffusive particle flux incorporating the effect of shear-induced particle migration in the directions of decreasing particle concentration and decreasing shear rate.

Resuspension of solid particles in shear flows is governed by two mechanisms which are considered in details in the following subsections.

Relative Motion Between Solid Particles and Fluid

The velocity of a solid particle species $\mathbf{u}_{si}$ can be defined from the mixture velocity $\mathbf{u}$ and the relative velocity between the particles and the carrier fluid. In the one-field approach, it is assumed that the time for a solid particle to reach its terminal settling velocity (relaxation time) is short and the solid particles are always moving at their terminal velocity. The relative velocity can then be defined from the balance of forces acting on the mixture components or by using the drift-flux approximation (see Rudman (1997)).

If the concentration of the solids is sufficiently high they may interact with each other and a *hindered settling* function is needed to account for these interactions.

In this work, with the assumption that the inertial terms can be ignored in the force balance equation, the relative velocity between solid particles of type *i* and carrier fluid is determined as follows:

$$\mathbf{u}_{ri} = \frac{1}{\rho(1-\phi)R_i(\phi)}\left[(\rho-\rho_{si})\nabla\cdot\tau_f\right], \quad (4)$$

where $R_i(\phi)$ is the hindered settling factor and ρ is the mixture density.

Effects of Shear on Particle Resuspension

The shear-induced particle diffusion plays a major role in the solids resuspension. According to Leighton and Acrivos (1987), shear-induced particle diffusion is caused by two mechanisms. The first source of particle diffusion arises from the random self-diffusion process (or random walk motion) in the presence of particle concentration and shear rate gradients, which is termed as the *tracer* diffusivity. Due to the presence of higher particle concentration on one side of a particle than on the other side, the particle experiences more interactions on the higher concentration side and drifts towards the region of lower concentration. Gradients in the shear field have a similar effect on the particles resulting in a particle migration towards the region of low shear. According to Phillips et al. (1992), the particle flux resulting from this displacement is found to be proportional to $a^2\phi\nabla(\phi\dot{\gamma})$.

Using the concept of 'particle migration potential' Shauly et al. (2000) obtained the following flux formulation for this type of particle migration in polydisperse flows:

$$\mathbf{J}_{1i} = K_1 a_i \bar{a}\phi\nabla(\phi_i\dot{\gamma}), \quad (5)$$

where ϕ is the total solid volume fraction; $\bar{a}$ is the volume-averaged particle radius; and K_1 is a proportionality constant of order unity.

In addition, particle diffusion is also caused by gradients of the total solid concentration. As mentioned in the work by Leighton and Acrivos (1987), the presence of gradients in the total solid concentration results in gradients in viscosity, which, in turn, cause interactions between solid particles and migration the particles from regions of high viscosity to low. The particle flux caused by this diffusion mechanism was formulated by Leighton et al. (1987) and Phillips et al. (1992) for monodisperse suspensions and recently by Shauly et al.(2000) for polydisperse suspensions:

$$\mathbf{J}_{2i} = K_2\bar{a}^2\dot{\gamma}\phi\phi_i\frac{1}{\mu}\frac{d\mu}{d\phi}\nabla\phi\,, \quad (6)$$

where K_2 is also a proportionality constant of order unity.

It is worth noting that in multidimensional shear flows, parameters K_1 and K_2 may vary from one direction to another and should be of tensorial form. However, such directional dependences are ignored in this work and the above parameters are assumed to be scalars.

In turbulent flows, the shear field is significantly different from that in laminar flows. However, the above described model of shear-induced particle diffusion can still be used. In addition, other forces associated with turbulent flows, such as fluctuating lift forces or turbulent dispersion forces, may also contribute to the solid particle dispersion and resuspension. The effect of turbulence on particle resuspension is investigated in a separate study and not included in this work.

Numerical Method

The resupension model is implemented based on the CFX-4 algebraic slip model (ASM). The CFX ASM has been enhanced for concentrated particulate flows. The additional diffusive fluxes are modelled by modifying the diffusion coefficients and source term in the particle transport equations.

PREDICTION RESULTS

In this section the developed computer model is applied to examine some experimental and analytical results reported in the literature.

Distribution of Solid Particles in A Sheared Narrow Gap

The model is first applied for predicting the settling of monodisperse suspension in a simple shear flow. The simulation conditions are set in accordance with the experiment by Leighton et al. (1986). In that experiment, the sheared region was an annular area confined between a top (static) plate and bottom (rotating) plate. The gap between the plates was 5 mm. Different particle sizes and suspending fluids were used in the experiment. In this simulation, the fluid carrier density and viscosity are assumed to be 1090 kg/m^3 and 3.4 p, respectively. The solid particles have a density of 2452 kg/m^3 and an average size of 139 μm. The rotation speed of the lower plate is varied to obtain different shear rates.

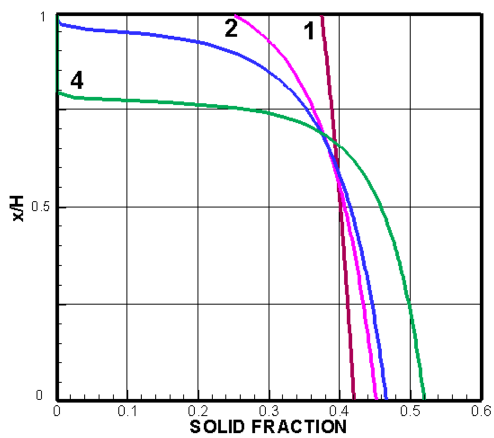

Figure 1. Vertical distributions of the solids at different shear rates – monodisperse suspensions – shear rate equal to 27.65, 8.74, 6.18, and 1.95 1/s in cases 1,2,3 and 4, respectively.

Distributions of the solid particles at different shear rates are shown in Figure 1. The computational results clearly demonstrate the effect of viscous resuspension which counters the settling under gravity. The above results are generally in good agreement with the experimental observations.

Viscous resuspension of bi- and, generally, poly-disperse suspensions is much more difficult to investigate experimentally. Few experimental studies of shear-induced diffusion in bidisperse suspensions have been reported in the literature (see, for example, Krishnan et al., 1995). However, in these studies, details about solids distributions were not presented and, as in the work by Krishnan et al. (1995), only information about the effective mixture viscosity was available. Some analytical studies have been carried out for viscous resuspension in polydisperse concentrated suspensions (Tripathi et al. (1999), Shauly et al. (2000)). Their results indicate the dependencies of viscous resuspension and possible particle segregation on particle sizes and densities.

Our numerical analysis of resuspension in bidisperse flows uses the flow and suspension parameters similar to those in Krisnan et al.'s experiment. As in the work by Leighton et al. (1986), the experiment by Krishnan et al. (1995) was conducted using an annular parallel plate viscometer. The suspension is comprised of a glycerin-water solution of 1230 kg/m^3 density and solid particles of two types, i.e. two different sizes, 51/119µm, and slightly different densities, 2340/2420kg/m^3. The computational result for a constant shear rate of 3.82s^{-1} is shown in Figure 2. Segregation of the solid particles is observed with small particles predominantly located in the upper part of the gap and large particles in the lower part. However, the transitions in the particle concentration profiles are smooth and a clear boundary between particles of different types is not seen. This computational result compares reasonably well with the analytical study by Shauly et al. (2000).

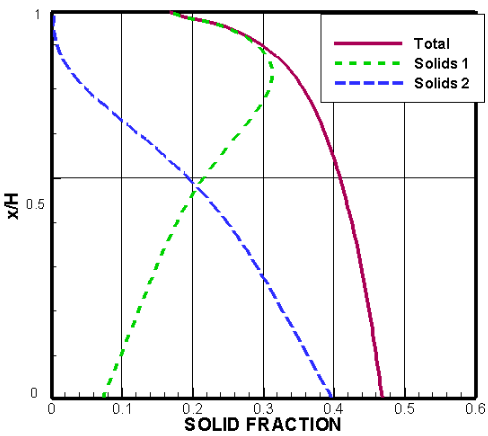

Figure 2. Distribution of the solid concentration across the gap.

Solid Concentration Profiles in Pipe Flows

In pipe flows, the shear rate is not constant, i.e. it is zero at the centreline and highest at the wall. Such a gradient in the shear field would result in a particle flux towards the pipe centreline.

In this section, the developed computer model is applied to predict solid concentration profiles in a pipe flow. The Reynolds number of the flow under consideration is 2000. The particle size is assumed to be 1.19mm and the solid-liquid density ratio is about 1.96.

The computation results for two mean solid concentrations are shown in Figures 3 and 4. In the figures, *Umax* is the maximum velocity at the inlet where the solid distribution is uniform. As can be seen from the figures, our model is able to predict the migration of the solid particles towards the pipe centreline where the shear rate is lowest. The computational results also demonstrate the flattening of velocity profile near the centre of the pipe and the dependence of magnitude of this flattening on the mean solid concentration. This behaviour of the velocity profile is in qualitative agreement with the results reported by Phillips et al. (1992).

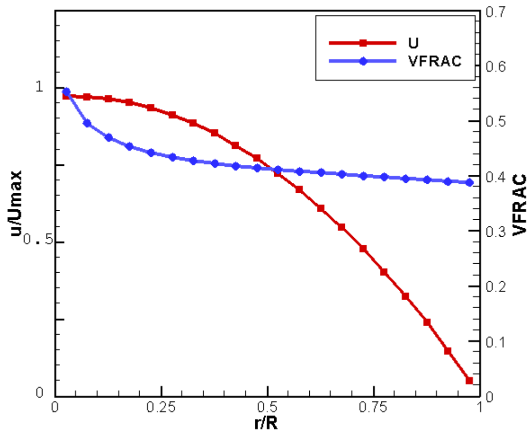

Figure 3. Profiles of the axial velocity and solid concentration – mean solid concentration is 0.4.

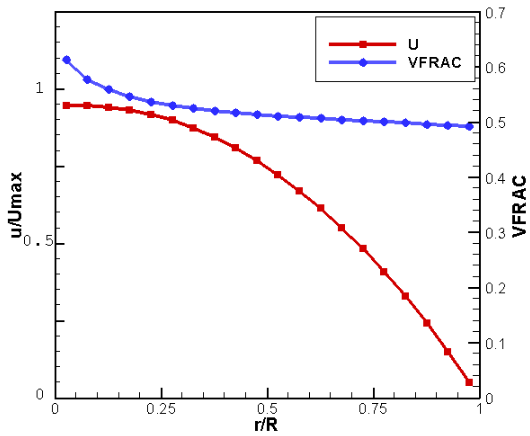

Figure 4. Profiles of the axial velocity and solid concentration – mean solid concentration is 0.5.

CONCLUSION

In this work, a computational model is developed to investigate the phenomenon of shear-induced particle migration and viscous resuspension. The model is based on the CFX-4 algebraic slip multiphase model and includes the descriptions of both hindered settling and shear-induced particle transport. Derived from the one-field description of multiphase flows, the model is highly effective and robust for polydisperse suspension flow simulation. Mechanisms affecting the transport of solid particles in shear flow are considered and included as closure correlations in the model. The model has been applied to predict solid concentration profiles in some typical flow conditions. For the pipe flows, the computational simulation correctly predicts the migration of the solid particles towards the regions of low shear. The effects of the shear rate and suspension composition on the solid distribution are also correctly described for monoand bi-disperse suspensions subjected to simple shear field.

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