

THE APPLICATION OF COMPUTATIONAL FLUID DYNAMICS AND HEAT TRANSFER IN MEASURING THE MAGNITUDE AND DIRECTION OF VELOCITY IN LIQUID METALS

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ABSTRACT

The sphere melting method represents a generic technique to measure velocity in liquid metals. This paper deals with the numerical simulation of the sphere melting technique. The melting time of a metal sphere immersed in a liquid metal of the same composition is related to the diameter and the velocity and temperature of the bath. This can be used to measure the magnitude of velocity in liquid metals. In addition a modification of this technique can detect the liquid metal direction.

The numerical model consists of a transient convective heat transfer and fluid flow system coupled by the presence of a moving boundary. The computer code used in this numerical model was developed using the SIMPLER algorithm in 3D. In addition, experimental verification work was carried out in the Aluminum and Magnesium systems. The findings of the experimental work are compared with the numerical predictions. A good agreement is obtained.

NOMENCLATURE

c	Heat capacity, $J/kg^{\circ}C$
D	Diameter of sphere, m
DT	Time interval used in numerical solution, s
g	Gravity, m/s^2
H	Enthalpy, J/kg
k	Thermal conductivity, $W/m^{\circ}C$
LF	Liquid fraction, kg/kg
LH	Latent heat, J/kg
MT	Total melting time of addition, s
Q	Source term for energy equation, W/m^3
SPH	Superheat ($SPH=T_{\infty}-T_m$), $^{\circ}C$
T	Temperature, $^{\circ}C$
u	Velocity, m/s

β	Coefficient of thermal expansion, $1/^{\circ}C$
κ	Partition coefficient, dimensionless
μ	Viscosity, $kg/m.s$
ρ	Density, kg/m^3

Subscripts

∞	Liquid metal condition far away from sphere
0	Initial condition
e	Eutectic composition of the alloy
m	Melting point (or liquidus point of the alloy)
s	Solvent material of the alloy

INTRODUCTION

The great majority of fluid mechanics problems in engineering are associated with the measurement of fluid flow. The need for velocity measurements has increased substantially with the proliferation of models aimed at predicting fluid flow characteristics, since they provide the necessary validation and tuning for the predictions.

The problem faced by liquid metal practitioners when attempts are made to measure velocity in high temperature liquid metals or slags is that these fluids are very hostile and cannot be treated as "common fluids", i.e. fluids that are transparent at low temperatures and do not possess corrosive properties.

The melting time of a sphere immersed in a bath of the same composition depends on the diameter of the sphere, the bath's temperature and velocity. Using this technique, bath velocity is inferred from the sphere's melting time and the bath's temperature, both of which parameters can be measured directly. In an effort to explore further the sphere melting technique, it is imperative to investigate the complex heat and momentum phenomena around a melting sphere.

MATHEMATICAL MODEL

From a computational point of view, the major problem in modelling a solidification phase change system is dealing with the latent heat evolution. In the present work, two

methods are applied to model the solidification and melting of the material.

For the pure Aluminum a numerical integration algorithm is used due to the fact that the phase change is isothermal. For the Magnesium alloy AZ91 a source term based method is used to account for the absorption of heat during solidification.

The properties of the materials used in the models can be seen in Table 1. The low values of the Prandtl number in these systems make them unique in the sense that there is very little information in the literature regarding analytical formulations, numerical predictions and even to a lesser extent, empirical correlations.

	k	ρ	c	μ	β	T_m	LH
Sol. Al.	220	2700	1000	-	-		
Liq. Al.	90	2400	1100	1.2×10^{-3}	1.3×10^{-4}	660	395,000
S. Steel	15	7800	500	-	-	-	-
Sol. AZ91	80	1750	1200	-	-		
Liq. AZ91	60	1650	1400	1.4×10^{-3}	1.2×10^{-4}	600	370,000

Table 1: Thermo-physical properties of the materials used in the models (in SI units).

The parameters involved in the predictions will be the temperature of the bath, expressed as the superheat ($SPH = T_\infty - T_m$), sphere diameter (D) and velocity of the bath relative to the sphere (u_∞)

The problem of a sphere melting in a bath is treated as a three-dimensional problem in Cartesian coordinates of transient heat transfer and fluid flow coupled by the presence of natural convection. A Control Volume method is used to solve the governing equations in three dimensions (SIMPLER algorithm; Patankar, 1980.) The thermal resistance around the solid sphere is included in the model. The Boussinesq approximation is used to model the buoyancy forces in the z -direction.

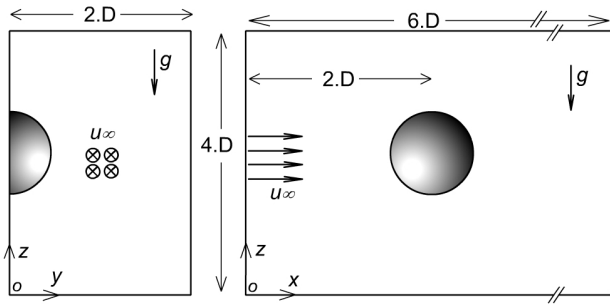


Figure 1: Schematic of domain used for the numerical simulation

The domain size used depends on the diameter of the addition. A minimum size of 6 diameters in the direction of the velocity (x) and 4 diameters in the directions perpendicular to the velocity (y, z) was found to be large enough to obtain a stable and accurate solution. Figure 1

shows a schematic of the domain. Only a half sphere is modelled due to the symmetrical nature of the system (symmetry plane $y = 0$).

The equations, boundary and initial condition are as follows:

Momentum equation in x direction

$$\rho \left(\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} \right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \mu \frac{\partial u_x}{\partial x} + \frac{\partial}{\partial y} \mu \frac{\partial u_x}{\partial y} + \frac{\partial}{\partial z} \mu \frac{\partial u_x}{\partial z}$$

Momentum equation in y direction

$$\rho \left(\frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} \right) = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \mu \frac{\partial u_y}{\partial x} + \frac{\partial}{\partial y} \mu \frac{\partial u_y}{\partial y} + \frac{\partial}{\partial z} \mu \frac{\partial u_y}{\partial z}$$

Momentum equation in z direction

$$\rho \left(\frac{\partial u_z}{\partial t} + u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x} \mu \frac{\partial u_z}{\partial x} + \frac{\partial}{\partial y} \mu \frac{\partial u_z}{\partial y} + \frac{\partial}{\partial z} \mu \frac{\partial u_z}{\partial z} - \rho \cdot g \cdot \beta \cdot (T - T_\infty)$$

Energy equation

$$\rho \cdot c \left(\frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} + u_z \frac{\partial T}{\partial z} \right) = \frac{\partial}{\partial x} k \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} k \frac{\partial T}{\partial y} + \frac{\partial}{\partial z} k \frac{\partial T}{\partial z} + Q$$

Boundary Conditions

Inlet ($x=0$), Bottom ($z=0$), Top ($z=4.D$) and Back ($y=2.D$):

$$u_x = u_\infty; u_y = u_z = 0; T = T_\infty$$

Outlet ($x=6.D$): $\partial u_x / \partial x = \partial u_y / \partial x = \partial u_z / \partial x = \partial T / \partial x = 0$

Front ($y=0$): $u_y = 0; \partial u_x / \partial y = \partial u_z / \partial y = \partial T / \partial y = 0$

Initial Condition

Fluid: Liquid at $T = T_\infty$

Sphere: Solid at $T = T_0, (T_0 < T_m < T_\infty)$

A structured mesh compatible with the SIMPLER algorithm was employed. Due to the three-dimensional character of the simulation, a non-uniform mesh was used to reduce the computational time without compromising the accuracy of the solution. An inter-nodal distance of $D/30$ is used in the sphere up to a radius of $0.75 \times D$, while for the rest of the domain the inter-nodal distance is $D/10$. The total control volumes are then 90 in the x -direction, 70 in the y -direction and 35 in the z -direction, totalling 2×10^5 control volumes. A detail of the mesh around the sphere can be seen in figure 2. Due to the high thermal gradients during the initial transient, a value of $10^{-5} (\rho \cdot c \cdot D^2 / k)$ is used during the first second after immersion; the term $(\rho \cdot c \cdot D^2 / k)$ being a representative time. A time interval ten times larger is used for the remaining

part of the simulation. This combination of time intervals and control volume distribution was found to be the best compromise between accuracy and computational time.

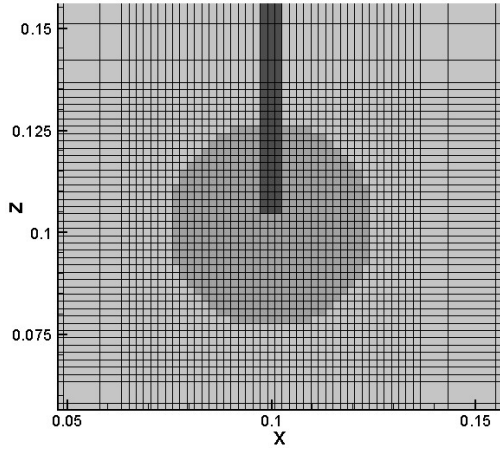


Figure 2. Non-uniform structured SIMPLER mesh around sphere (x - z symmetry plane).

The spheres are immersed in the bath by means of a 6 mm diameter stainless steel tube. For this reason, the holder is introduced in the model as depicted in figure 2. The thermo-physical properties of the stainless steel used can be found in Table 1. An interfacial heat resistance value of 10^{-4} m²K/W is used around the sphere due to the oxide layer formed (Argyropoulos *et al.*, 1999.)

The solid material is modeled by forcing the viscosity to increase 20 orders of magnitude during the change of phase (Fisher, 1986.)

Phase change boundary tracking in Aluminum

The Numerical Integration Algorithm is used due to the fact that the phase change is modeled as isothermal. When a pure material is melting/solidifying, the heat supplied/extracted is used for the phase change and there is no temperature change. In the model, when a control volume begins melting/solidifying, its temperature is set at the melting point until the amount of heat supplied/extracted accounts for the latent heat of fusion.

Phase change boundary tracking in AZ91

Due to the large freezing range for the AZ91 alloy, the numerical integration algorithm is not applicable. The latent heat evolution in alloys is controlled by the nature of the local liquid fraction. Although this variable could be a function of a number of variables, for most practical systems it can be assumed to be a function of temperature alone, i.e.: $LF = f(T)$. A physically meaningful liquid fraction relationship follows from the Scheil equation (Fisher, 1986.)

$$LF = \left(\frac{T_{m,s} - T}{T_{m,s} - T_m} \right)^{\frac{1}{1-\kappa}} \quad \{T_e \leq T \leq T_m\}$$

The partition coefficient $\kappa = 0.37$ is extracted from the Mg-Al phase diagram.

Figure 3 shows the relationship between liquid fraction and temperature from the Scheil equation for AZ91. The eutectic temperature for this alloy is 437°C.

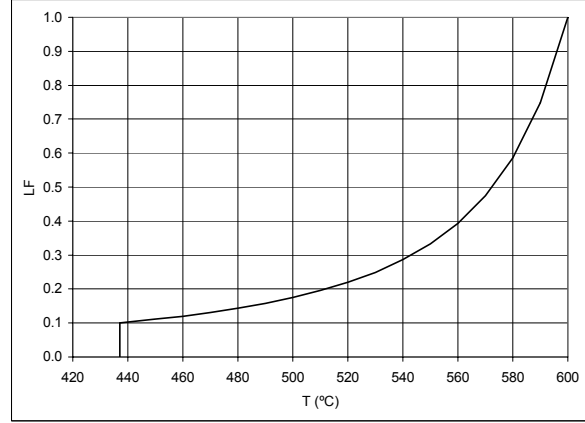


Figure 3. Relationship between liquid fraction and temperature for AZ91 according to the Scheil equation.

To model the phase change, a source term is introduced to account for the absorption/release of heat during melting/solidification.

$$\begin{aligned} \partial H &= c \cdot \partial T + LH \cdot \partial LF \\ \Rightarrow \rho \frac{\partial H}{\partial t} &= \rho \frac{c \cdot \partial T}{\partial t} + \rho \cdot LH \cdot \frac{\partial LF}{\partial t} \\ \Rightarrow Q &= -\rho \cdot LH \frac{\partial LF}{\partial t} \end{aligned}$$

Results of the Numerical Simulation

Figures 4, 5 and 6 show the velocity vectors and isotherm plots from the numerical simulation of a 5 cm Aluminum sphere immersed in a 720°C bath at a velocity u_∞ of 0.2 m/s. The shape of the original sphere and holder are shown for comparison. Due to the high conductivity of the solid phase, a solid shell quickly forms (figure 4). One second after immersion, the sphere exceeds 300°C at almost every point, while the shell has melted back in its stagnation point (figure 5). After 5 seconds, melting has almost reached 50% of the radius at the stagnation point (figure 6). When the centre of the sphere becomes liquid, the simulation stops.

Figures 7 and 8 show the velocity vectors and isotherm plots for a 5 cm AZ91 sphere in a 630°C bath and a velocity of 0.1m/s. The thermo-physical properties of Aluminum and AZ91 are of the same order, so the melting dynamics and total time is similar.

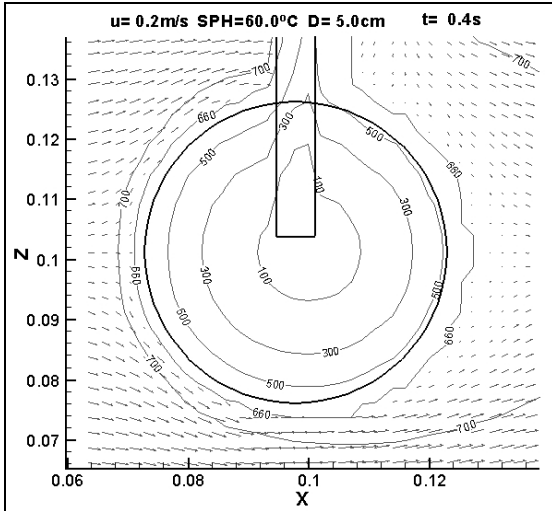


Figure 4: Aluminum Sphere. Time = 0.4 s

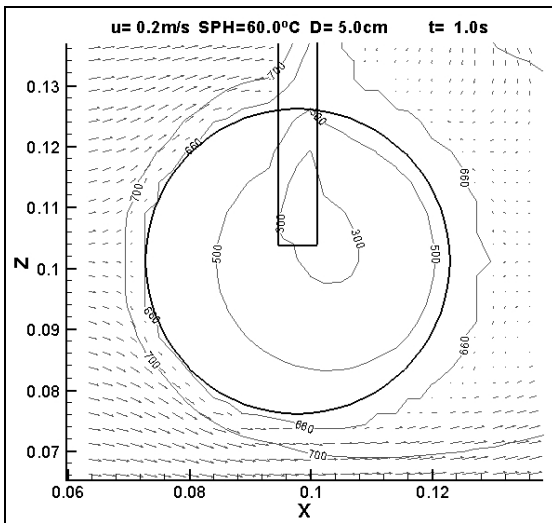


Figure 5: Aluminum Sphere. Time = 1.0 s

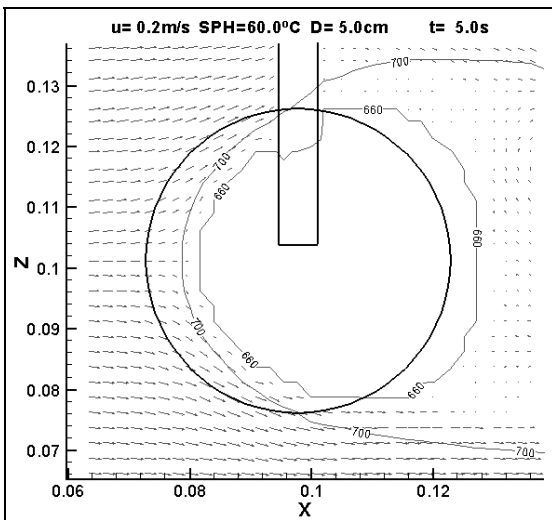


Figure 6: Aluminum Sphere. Time = 5.0 s

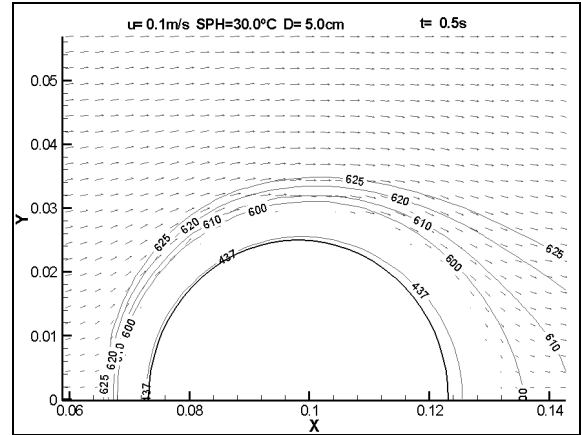


Figure 7: AZ91 sphere. Time = 0.5 s

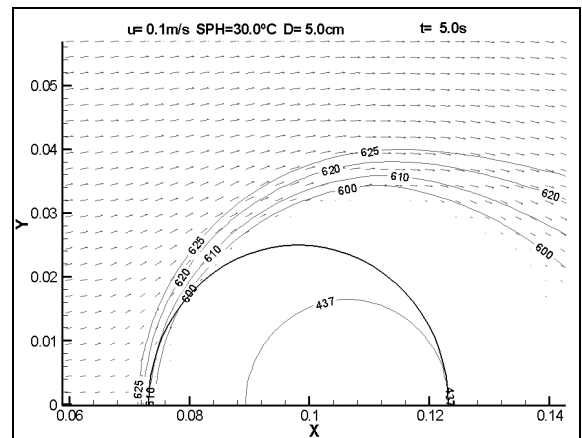


Figure 8: AZ91 sphere. Time = 5.0 s

Experimental Results

The experimental apparatus consists of a tank revolving inside a heavily insulated electrical resistance furnace (Revolving Liquid Metal Tank). The temperature and velocity of the tank can be controlled, while the spheres are immersed from the top. The melting times of the spheres are measured and compared with the numerical modelling results. The experiments in AZ91 are done under a controlled atmosphere of inert gas to prevent the reaction of the Magnesium alloy. Figure 9 shows a schematic of the experimental set-up and sphere-holder assembly.

Figures 10 and 11 show a comparison between the numerical predictions and the experimental results for Aluminum spheres of diameters of 5cm and 7cm respectively. Figure 12 shows the comparison for 5cm AZ91 spheres in a 630°C bath. The standard deviation of the melting times for both systems in superheats ranging from 30°C to 60°C is approximately 10%.

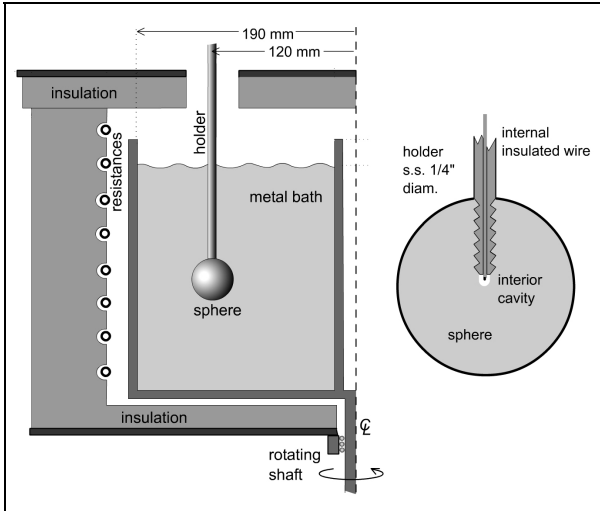


Figure 9: Schematic of RLMT and sphere-holder assembly.

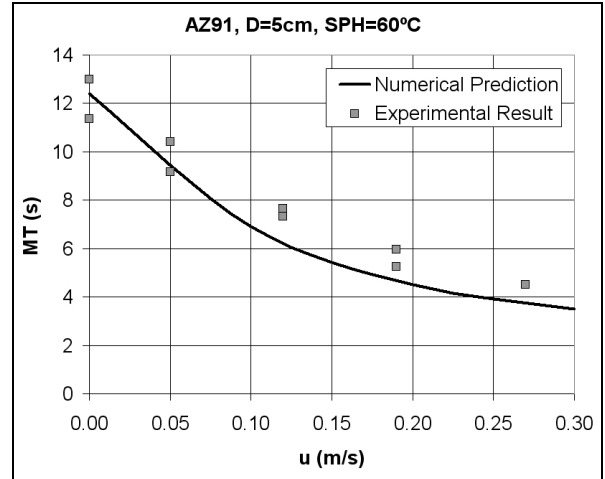


Figure 12: Experimental points and numerical results for 5 cm AZ91 spheres in 660°C bath.

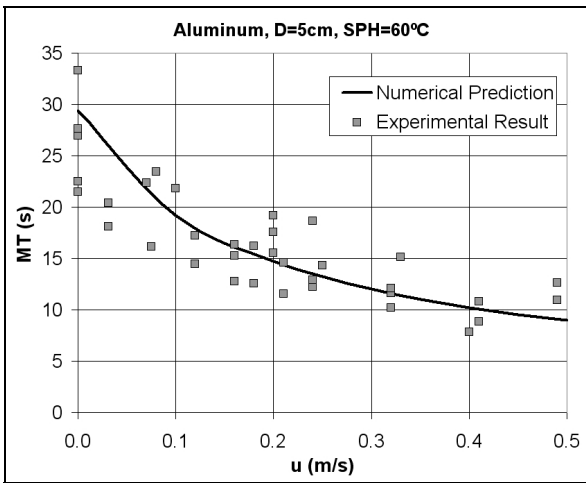


Figure 10: Experimental points and numerical results for 5 cm Aluminum spheres in 720°C bath.

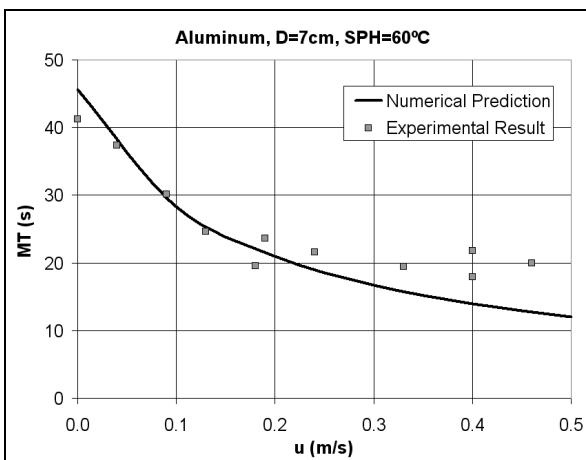


Figure 11: Experimental points and numerical results for 7 cm spheres in 720°C bath.

Direction of velocity in liquid metals

The location that has the highest melting rate within the sphere is the stagnation point, as can be seen in figures 4 through 8. By placing several melting probes in a single sphere it is possible to determine the direction of the velocity in a liquid metal.

Figure 13 shows the schematic of a 5cm Aluminum sphere with three melting probes at 120° from each other in a horizontal plane through the centre of the sphere. Several trials were done at superheats between 30°C and 60°C at velocities from 0 to 0.2m/s.

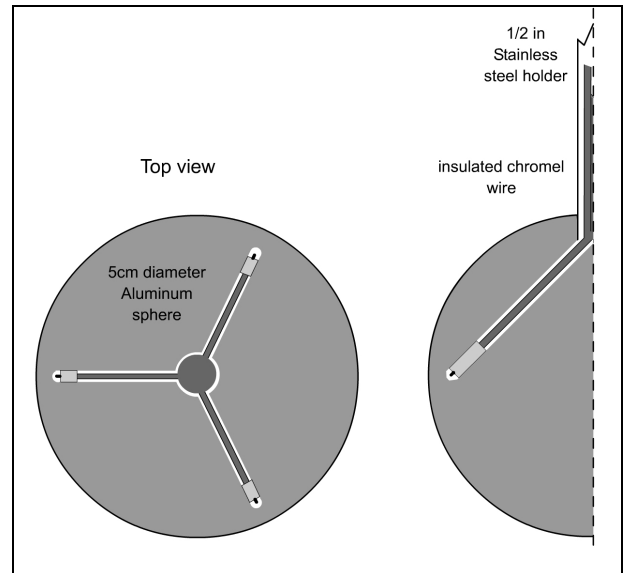


Figure 13: Schematic of direction of velocity probe.

Figure 14 shows an example of the data acquired with a 5cm sphere and a bath of superheat of 30°C and a velocity of 0.1m/s. The first probe to detect the melting is the one located in the stagnation point of the flow. The other two

probes melt much later than the first one and have almost identical melting times, as it is expected from symmetry.

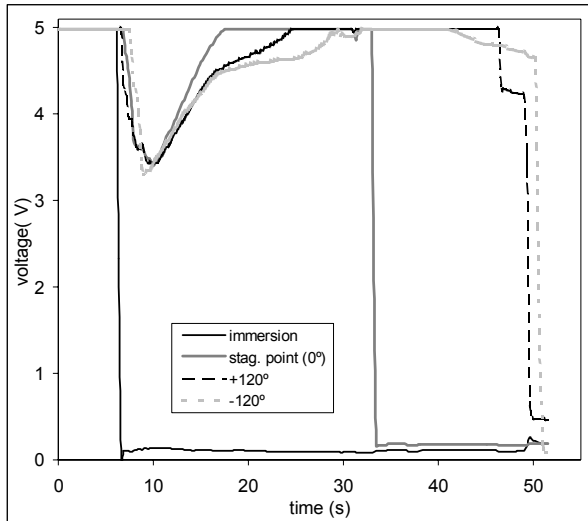


Figure 14: Data acquired for a 5 cm sphere in an Aluminum bath with a velocity of 10 cm/s, SPH=30°C.

CONCLUSIONS

Melting times of Aluminum spheres immersed in an Aluminum bath have been evaluated using a mathematical model and an experimental set-up. Good agreement was obtained between the numerical results and the experimental data points for the whole velocity range for the case studied. If the bath velocity were to be inferred from the measurement of the melting time of an Aluminum sphere, an error of around ± 10 cm/s would be incurred. A possible way of decreasing the error involved in the measurement is to immerse several spheres sequentially. With three sequential immersions the error will be almost halved.

A method of detecting direction of velocity in liquid metals is proposed. The experimental results with three probes show promising results.

ACKNOWLEDGEMENTS

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