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DNS OF TURBULENT NON-NEWTONIAN FLOW IN AN OPEN CHANNEL

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ABSTRACT

The transport of non-Newtonian, fine-particle slurries in open channels is common practice in tailings disposal application in the mining industry, as are the self-formed channels observed in CTD (Central Tailings Discharge) operations. Direct numerical simulation (DNS) of the turbulent flow of non-Newtonian fluids in an open channel are modelled using a spectral element-Fourier method. The simulation of a yield–pseudoplastic fluid using the Herschel-Bulkley model agrees qualitatively with experimental results from field measurements of mineral tailing slurries. The simulation results over–predict the flow velocity by approximately 15% for the cases considered, although the source of the discrepancy is difficult to ascertain. The effect of variation in yield stress and assumed flow depth are investigated and used to assess the sensitivity of the flow to these physical parameters. This methodology shows some potential in designing and optimising the transport of slurries in open channels.

NOMENCLATURE

A	Area
K	Fluid consistency index
n	Flow index
r	Radius
P	Wetted perimeter
p	Pressure
Re	Reynolds number
r_H	Hydraulic radius
S	Slope
V	Average velocity
U_*	Friction velocity
y+	Distance from the wall, wall units
ρ	Density
γ	Shear rate
η	Dynamic Viscosity
η_w	Mean wall Viscosity
τ	Shear stress
τ_w	Wall shear stress
τ_y	Yield stress
θ	Segment angle

INTRODUCTION

The flow of non-Newtonian fluids in open channels is of great importance to the mining industry. Fixed-shape open channels can transport ore slurries and tailings streams as a more economic alternative to pumping, when the terrain allows. Currently the design of these flumes is often done using crude estimates based on the conditions established for water with a limited set of field observations (Wilson, 1991). In the context of a tailings storage facility, self-formed channels at sufficient gradient or slope will generate enough turbulence to maintain particles in suspension. Also, a shallower gradient will reduce the turbulence intensity, which allows more solids to settle in the channel bed. The study of non-Newtonian suspensions in open channels will provide the fundamental information allow the design and operation of industrial channels for the transport of mineral suspensions.

Experimental measurement of velocity profiles and turbulence statistics in the laboratory in suitable model fluids can be used to determine the important characteristics of turbulent channel flow of a non-Newtonian fluid. If the rheology of the suspension is known, then computational simulation can be used to simulate the flow. Computational modelling of non-Newtonian fluids using direct numerical simulation (DNS) shows at least qualitative agreement when compared with reality and sometimes reasonable quantitative agreement has been found also in Rudman and Blackburn (2006) and Rudman *et al* (2004). The main benefit of using a DNS technique is that once it is validated, it can be used to model flow behaviour and provide a detailed understanding of the turbulent structure and the associated potential for particle transportation in channels.

In 1987, Kim, Moin and Moser applied Direct Numerical Simulation in order to test fully developed turbulent flow between two parallel plates. Rudman and Blackburn (2003) and Rudman and Blackburn (2004) have used spectral element method to simulate non-Newtonian flow in pipes. The results obtained when a Herschel-Bulkley model was used showed good agreement in terms of shape and magnitude when compare with the experimental data. The objective of this paper is to describe a study of shear-thinning non-Newtonian fluids flowing in an open flume whose rheology can be described using the Herschel-Bulkley model.

Rheology

In the current work, the viscosity η can be described using the Herschel – Bulkley model

$$\eta = \frac{\tau_Y}{\dot{\gamma}} + K\dot{\gamma}^{n-1} \qquad (1)$$

Where K is the fluid consistency index, n is the flow behaviour index, and τ_y is the fluid yield stress.

Wall viscosity

The mean wall viscosity η_w is calculated from the mean wall shear stress, τ_w.

$$\tau_w = \frac{\delta p}{\delta z} \rho \frac{A}{R\theta} \qquad (2)$$

ssuming a Herschel-Bulkley rheology,

$$\eta_w = K^{1/n} \frac{\tau_w}{(\tau_w - \tau_y)^{1/n}} \qquad (3)$$

Wall units

Wall units are introduced with the wall viscosity instead of the non-Newtonian viscosity. Therefore the friction velocity is defined as $U^* = \sqrt{\tau_w / \rho}$, the non-dimensional velocity is $U^+ = U / U^*$ and the non-dimensional distance from the wall is written $y^+ = (\rho U^* / \eta_w) y$.

EXPERIMENTAL WORK

The experimental work was conducted at the Sunrise Dam Gold Mine, in Western Australia. The flume was semi-circular in cross section with an internal diameter of 340 mm. A plunge box was located at the upstream end of the flume. The tailings slurry was supplied to the plunge box through a 150 mm High Density Polyethylene (HDPE) pipe with an outlet 20 cm above the plunge box floor. A diaphragm valve was installed in this pipe to allow adjustment of the flow rate of tailings slurry.

At the downstream end of the flume, a flow measuring box was placed to measure the flow rate of the flume. A level indicator and a stopwatch were used to record the time taken for the flume discharge to fill a specific volume. Local velocities at specific locations within the channel were measured using a Delft E-30 velocity probe, which generates an electromagnetic field from 5 electrodes mounted in the bottom surface of an ellipsoid head with 33 mm in diameter and 11 mm thick. Due to the requirement for all five electrodes to be immersed in the fluid without any air interaction at the surface, it was found that the E-30 probe could not measure fluid velocities at depths less than 5 mm. It was discovered that the width of the ellipsoid head of the probe prevented velocity measurements from being taken within about 17 mm of the boundaries of the half pipe. Further details of this experiment and associated instrumentation are given in Jewell and Lawson (2006).

Rheometric analyses were performed on samples of the slurries at a number of concentrations and the rheological parameters were deteremined by fitting the data to the Herschel-Bulkley rheological model. The rheometric measurements were done with a Contraves Rheomat 115 Couette rheometer, using a 45 mm diameter bob with vertical grooves to reduce slip. The rheological model is then fitted to the rheograms. The rheological parameters based on this experimental data were used in the simulation.

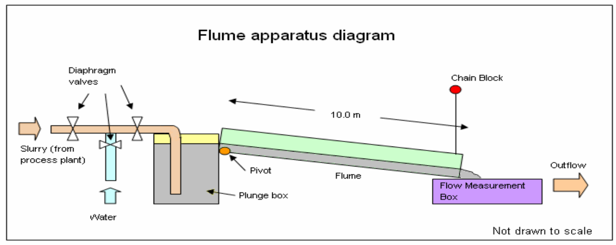

Figure 1 Flume apparatus diagram

NUMERICAL METHOD

The numerical method is based on spectral element/Fourier formulation. The three-dimensional spatial discretisation uses isoparametrically mapped quadrilateral spectral elements in the x-y plane, allowing arbitrary geometry and a Fourier discretization in the z (out-of-plane) direction. The z-axis is aligned with the flow direction.

The governing equations for the three-dimensional numerical simulation are the incompressible Navier-Stokes equations

$$\frac{\partial u}{\partial t} + N(u) = -\nabla P + \nu \nabla^2 u \qquad (4)$$

$$\nabla \bullet u = 0 \qquad (5)$$

Where P = p/ρ and N(u) represents the non-linear terms.

Details of the numerical method may be found in Rudman and Blackburn (2006) and are not repeated here in the interests of space.

In order to drive the flow in the axial (z) direction, a body force per unit mass equivalent to the slope measured in the experiments is applied to the z-momentum equation.

A summary of simulation parameters is presented in Table 1.

Table 1 Parameters for simulation

Model Hershel-Bulkley	**n**	**K [Pa.s^n]**	**τ_y [Pa]**	**δp/δz**	**Length**	**Modes**
Simulation parameter based on experiment						
	0.81	0.0506	2.249	0.147	0.5πd	384

The computations reported here were carried out using 16-32 processors on a supercomputer from VPAC.

EXPERIMENTAL RESULTS

Velocity fluctuation and point velocity of the slurry were measured. The velocity is then non-dimensionalised using the wall viscosity. The wall viscosity is calculated by wall shear stress. The wall shear stress is found from:

$$\tau_w = g \cdot \rho \cdot \sin\theta \cdot r_H \qquad (6)$$

Where

$$r_H = \frac{A}{P} \qquad (7)$$

Clapp (1961) reports the results of experimental measurements of the turbulent pipe flow of power law fluids with flow indices in the range of 0.698-0.813. Clapp determines that the logarithmic velocity profile for the turbulent flow of power law fluids is a function of the flow index, *n*, and satisfies

$$U^+ = \frac{A}{n} + \frac{B}{n} \ln y^+ \qquad (8)$$

Where

$$y^+ = \left[\frac{\left(\rho^n \tau_w^{\ 2-n}\right)^{1/2}}{K} \right] y^n \qquad (9)$$

The value of these coefficients for well-developed turbulent flow of Newtonian fluids (where *n* equals to 0) are now generally accepted to be A= 5.5, B=2.5. However in this case, the logarithmic profile used for all plots is

$$U^+ = 5.5 + 2.5 \ln y^+ \qquad (10)$$

The mean axial velocity data at the centreline for the slurry is presented in Figure 2, in wall units, together with the logarithmic velocity profile. The experimentally measured velocity profile has a linear relationship between U^+ and y^+ in the near wall region. In the logarithmic region, the actual velocity profile for the slurry is slightly higher than the logarithmic velocity profile. At the free surface region, the measured velocity profile declined after a maximum. According to Nezu and Nakagawa (1993) and Nezu (2005), this could be concluded as a secondary flow effect where the lower velocity/momentum material has been transported into the high velocity/momentum region. Therefore there is a dip in the axial velocity profile after maximum velocity has been reached. This happens regularly in non circular channel flow. It is also suggested by Nezu and Nakagawa (1993) that this phenomenon is called the "velocity-dip", and it is peculiar to open channel flows.

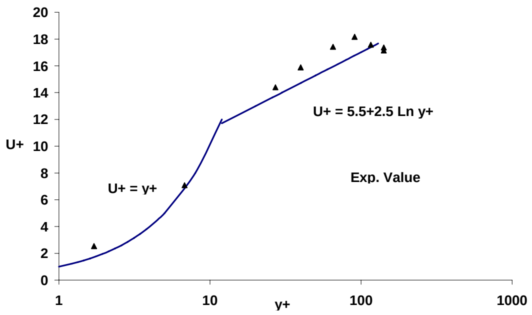

Figure 2 Experimentally measured velocity profile for slurry.

NUMERICAL RESULTS

The computationally predicted profile for Simulation 1 is presented in Figure 3. DNS turbulent pipe flow data from Rudman and Blackburn (2006) is also presented in the same plot. The velocities have been non-dimensionalised. The non-dimensionalisation is undertaken using the wall viscosity given in equation 3. The channel profile is also in good agreement with accepted profile for turbulent pipe flow. All profiles have a linear relationship between U^+ and y^+ in the near wall region. In the logarithmic region, the simulation profile and Rudman and Blackburn (2006) profile is above logarithmic velocity profile. This is consistent with experimental results.

However, there is a discrepancy between both simulation data and experimental data. The simulation profile and experimental profile have the same magnitude in the near wall region. In the free surface region, the simulation profile does not show any secondary flow effect. At this stage, there is no clear understanding of why this is the case. Figure 4 shows the instantaneous point velocity at the centre line of the channel. Both plots demonstrate a similar pattern of axial velocity. Thus it is suggested that simulation and field experiment are at the same magnitude and it provided some agreement between the two results.

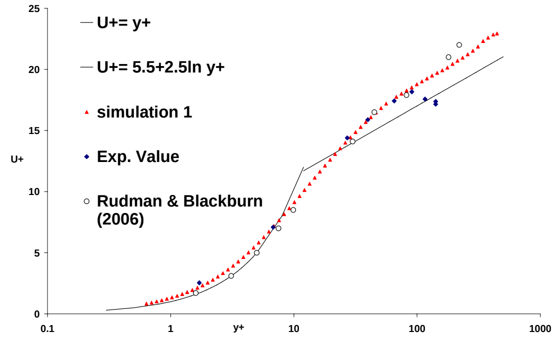

Figure 3 Experimentally measured velocity profile in conventional wall units for slurry in comparison of Simulation results.

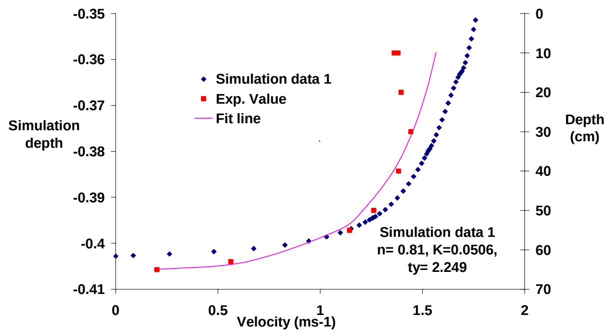

Figure 4 Experimentally measured velocity profile at centre line for slurry in comparison of Simulation results.

Cross-sectional velocities are shown in Figure 5. The contour shows a higher axial velocity at the centre and top region of the channel, and lower axial velocity at the bottom and edge of the channel. From the velocity vectors, it is observed that the flow pattern is turbulent. The turbulent eddies are detected on the bottom of the channel.

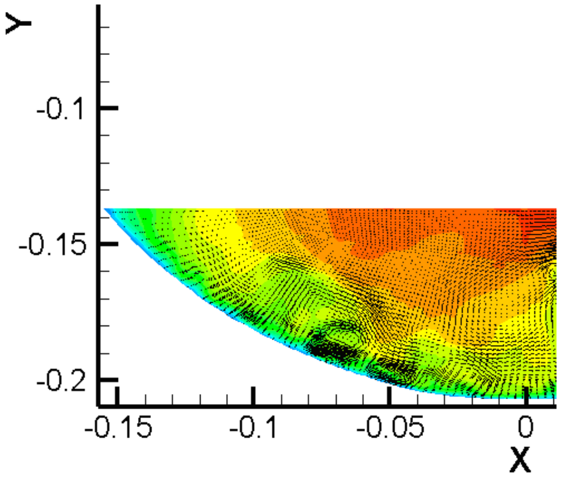

Figure 5 Contours of axial velocity and in-plane velocity vectors.

Contours of streamwise velocity at the wall are shown in Figure 6. The current simulation's contour is presented on the right hand side and Rudman *et al* (2001)'s contour is presented on left hand side. The wall streaks seen in current simulation is not as long as Rudman contour's wall streaks. The current simulation's contour is suggestive of fully developed turbulence.

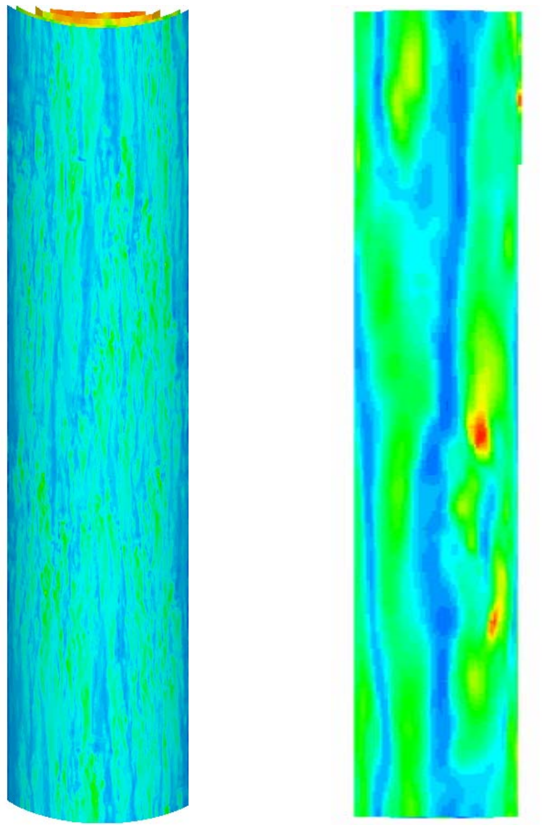

Figure 6 Near wall structure revealed in contours of streamwise velocity at wall. Re = 7848 (left) and Re =3964 (right)

CONCLUSION

The simulation results for a shear thinning fluid with Herschel-Bulkley model show some agreement as well as some significant differences with the experimental results. They showed similar behaviour in the near wall region; however the log-law profile lay above the theoretical profile. Validation has been done by Rudman and Blackburn (2003) on the numerical method in turbulent Newtonian flow and laminar flows of power-law fluids, and in both cases reasonable agreements were found. However, there are possibilities that fundamental errors in the numerical method for non-Newtonian channel flow exist. It is also possible that the resolution of the computation is not enough. Further study is being undertaken to address this issue.

Another possible error is the domain length effects. It is suggested by Rudman *et al* (2001) that a longer domain length could well improve the result. The application of DNS to flows of non-Newtonian fluids with different rheological parameters can provide a better and correct understanding of effect of rheological parameters.

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