NUMERICAL SIMULATION OF HEAT TRANSFER OF NANOFLUIDS IN AN ENCLOSURE

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ABSTRACT
In this paper, the heat transfer and fluid flow due to buoyancy force in a square enclosure using nanofluids are studied. Four different types of model from the literature are considered for the effective viscosity of the nanofluid. Finite Element method has been applied to incorporate a homogeneous solid-liquid mixture formulation for the two-dimensional buoyancy-driven convection in the enclosure filled with three different types of nanofluids. Simulations have been carried out to investigate the effects of the volume fraction, Nusselt number and Grashof number. An increase in Nusselt number was found with the volume fraction of nanoparticles for the whole range of Grashof number. Other than the thermal conductivity, the effective dynamic viscosity found to play a major role in heat transfer enhancement as the significant difference is observed from different adopted models.

Keywords: Nanofluids; Natural convection; Finite Element Method; Enclosure

NOMENCLATURE

\( \beta_f \) Fluid thermal expansion coefficient
\( \beta_s \) Solid expansion coefficient
\( \phi \) Solid volume fraction
\( \nu_f \) Kinematic viscosity
\( \theta \) Dimensionless temperature,
\( \omega \) Vorticity
\( \Omega \) Dimensionless vorticity
\( \psi \) Stream function
\( \psi^* \) Dimensionless stream function
\( \rho \) Density
\( \mu \) Dynamic viscosity

Subscripts

\( \text{eff} \) Effective
\( f \) Fluid
\( H \) Hot
\( L \) Cold
\( nf \) Nanofluid
\( o \) Reference value
\( s \) Solid

INTRODUCTION
Heat transfer fluids provide an environment for adding or removing energy to systems and their efficiencies depend on their physical properties such as thermal conductivity, viscosity, density and heat capacity. Low thermal conductivity is often the primary limitation for heat transfer fluids such as water, oil, ethylene glycol mixture in enhancing the performance and the compactness of many engineering electronic devices. To overcome this drawback there is a strong motivation to develop advanced heat transfer fluids with substantially higher conductivities to enhance thermal characteristics, suspension of colloidal particles dubbed as nanofluids. Choi (1995) in his pioneered work uses small amount of particles which are dispersed into water and other fluids. It has since then been shown experimentally by many scientists and engineers. Lee et al. (1999), Xuan et al. (2000) and Eastman et al. (2001) used nanoparticles as Oxide, Copper and Alumina particles respectively and conclude that nanofluids can have anomalously higher thermal conductivities than that
of the base fluid, thus posing as a promising alternative for thermal applications.

In the literature little work has been done on natural convection phenomena in nanofluids with differentially heated enclosures. Hwang et al. (2007) investigated the buoyancy-driven heat transfer of water-based Al₂O₃ nanofluids in a rectangular cavity. They showed that the ratio of heat transfer coefficient of nanofluids to that of base fluid is decreased as the size of nanoparticles increases, or the average temperature of nanofluids is decreased. Khanfar et al. (2003) was the first to investigate the problem of buoyancy driven heat transfer enhancement of nanofluids in a two-dimensional enclosure. They tested different models for nanofluid density, viscosity, and thermal expansion coefficients and found that the suspended nanoparticles substantially increase the heat transfer rate any given Grashof number. Jou and Tzeng (2007) analyzed the heat transfer coefficient. Jang and Choi (2004) investigated the fraction of nanofluıds cause an increase in the average heat transfer rate any given Grashof number. Einstein’s model (1956)

\[ 
\mu_{eff} = \mu_f (1 + 2.5\phi), \quad \text{for } \phi < 0.05.
\]

(5a)

Brinkmann (1952) as

\[ 
\mu_{eff} = \frac{\mu_f}{(1 - \phi)^{0.5}}.
\]

(5b)

Brownian motion effect’s model (2005)

\[ 
\mu_{eff} = \mu_f (1 + 2.5\phi + 6.17\phi^2).
\]

(5c)

Pak and Cho’s Correlation (1998)

\[ 
\mu_{eff} = \mu_f (1 + 39.11\phi + 533.9\phi^2).
\]

(5d)

The heat capacitance of the nanofluid can be presented as

\[ 
(p_c \rho)_nf = (1 - \phi) (p_c \rho)_f + \phi (p_c \rho)_s.
\]

(5e)

The effective stagnant thermal conductivity of the solid liquid mixture was introduced by Wasp as follows

\[ 
\left( \frac{k_{eff}}{k_f} \right)_{stagant} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_f}.
\]

(5f)

\[ 
\begin{align*}

\left( \frac{k_{eff}}{k_f} \right)_{stagant} & = 1 + 2k_f + \phi(k_f - k_s) - k_f,

\end{align*}
\]

(5g)

\[ 
\frac{k_{eff}}{k_f} = \left( \frac{k_{eff}}{k_f} \right)_{stagant} + k_d.
\]

(5h)

\[ 
\frac{k_{eff}}{k_f} = C \left( \frac{p_c \rho}_{nf} \right) \left| \frac{\Omega}{1 - \phi} \right|.
\]

(5i)

The effective viscosity of a fluid of viscosity \( \mu_f \) containing a dilute suspension of small rigid spherical particles is given by four models.

\[ 
\mu_{eff} = \mu_f (1 + 2.5\phi), \quad \text{for } \phi < 0.05.
\]

(5j)

Finally in Section 5, we provide a conclusion on results obtained.

**MATHEMATIC FORMULATION:**

The geometry under consideration is a horizontal enclosure of height \( H \) and length \( L \). It is assumed that the third dimension of the cavity is large enough so that the fluid flow and heat transfer can be considered two dimensions. The vertical walls of the enclosure are subjected to temperature \( T_H \) and \( T_L \) at the vertical left and right walls, respectively while the adiabatic boundary conditions are applied at upper and horizontal walls. The fluid in the enclosure is a water based nanofluid containing different type of nanoparticles: Cu, Al₂O₃, and TiO₂. The nanofluid is assumed incompressible and the flow is assumed to be laminar. It is assumed that the base fluid (i.e. water) and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermophysical properties of the nanofluid are given in Table 1.

The governing equations for the present study in terms of the stream function-vorticity formulation are of the following form:

**Kinematics Equation**

\[ 
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\phi
\]

(1)

**Energy Equation**

\[ 
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \frac{\partial}{\partial x} \left[ \alpha_f + k_f \left( \frac{\partial \phi}{\partial x} \right) \right] \frac{\partial^2 T}{\partial x^2} + \frac{\partial}{\partial y} \left[ \alpha_f + k_f \left( \frac{\partial \phi}{\partial y} \right) \right] \frac{\partial^2 T}{\partial y^2}
\]

(2)

**Vorticity Equation**

\[ 
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial}{\partial x} \left[ \alpha_f + k_f \left( \frac{\partial \phi}{\partial x} \right) \right] \frac{\partial \psi}{\partial x} + \frac{\partial}{\partial y} \left[ \alpha_f + k_f \left( \frac{\partial \phi}{\partial y} \right) \right] \frac{\partial \psi}{\partial y}
\]

(3)

where \( \alpha_{nf} = \left( \frac{k_{eff}}{k_f} \right)_{stagant} \).

The effective density of a fluid containing suspended particles at a reference temperature is given by

\[ 
\rho_{nf, o} = (1 - \phi) \rho_{f, o} + \phi \rho_{s, o}.
\]

(4)

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\left( \frac{k_{eff}}{k_f} \right)_{stagant} = 1 + 2k_f + \phi(k_f - k_s) - k_f.
\]

(5g)

\[ 
\frac{k_{eff}}{k_f} = \left( \frac{k_{eff}}{k_f} \right)_{stagant} + k_d.
\]

(5h)

\[ 
\frac{k_{eff}}{k_f} = C \left( \frac{p_c \rho}_{nf} \right) \left| \frac{\Omega}{1 - \phi} \right|.
\]

(5i)
\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Pr} \sqrt{Gr} \left[ \frac{\partial}{\partial X} \left( \frac{\partial \theta}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \frac{\partial \theta}{\partial Y} \right) \right].
\]

\[
X = \left[ \frac{\beta_y}{\beta_x} \right] \frac{k_x}{k_y} + C \frac{d}{\mu} \sqrt{Gr} \sqrt{U^2 + V^2}
\]

where \( Gr = \frac{g \beta_f \Delta T H^3}{\nu^2} \), \( Pr = \frac{V_f}{\alpha_f} \), and \( A = \frac{L}{H} \), which is assumed unity in the investigation. The diameter of the nanoparticle \( d_f \) is taken as 10 nm in the present study. The physical dimension of the enclosure \( H \) is chosen to be 1 cm. The coefficient \( \lambda \) that appears next to the buoyancy term is given as
\[
\lambda = \frac{1}{1 + \phi \rho_s \rho_f} = \frac{1}{1 + \phi \rho_s \rho_f} \frac{\beta_y}{\beta_x} \frac{k_x}{k_y}\beta_x.
\]

The Boundary conditions are as follows:
\[
U = V = \psi = \frac{\partial \theta}{\partial Y} = 0, \quad \Omega = -\frac{\partial^2 \psi}{\partial X^2},
\]

at \( Y = 0,1 \) and \( 0 \leq X \leq 1 \)

\[
U = V = \psi = 0, \quad \theta = 0.5, \quad \Omega = -\frac{\partial^2 \psi}{\partial X^2},
\]

at \( X = 0,0 \leq Y \leq 1 \)

\[
U = V = \psi = 0, \quad \theta = -0.5, \quad \Omega = -\frac{\partial^2 \psi}{\partial X^2},
\]

at \( X = 1,0 \leq Y \leq 1 \)

Lots of factors such as thermal conductivity, heat capacitance of both the pure fluid and ultrasound particles, the volume fraction of the suspended particles, the dimension of these particles, flow structure and the viscosity are affecting the Nusselt number of nanofluid. The local variation of the Nusselt number are given by
\[
N u = \frac{Q}{Q_{refd}} = \frac{1}{1 + \phi \rho_s \rho_f} \frac{\beta_y}{\beta_x} \frac{k_x}{k_y} \left| \frac{\partial T}{\partial X} \right|_{x=0},
\]

where \( Q = -\left( k_{eff} \right)_{refd} A \frac{\partial T}{\partial X} \).  

### Table 1: Thermophysical properties of fluid and Nanoparticles

| Physical Properties | Fluid phase (water) | Cu | \( \lambda_{1,0,3} \) | TiO
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>( C_p ) (J/kgK)</td>
<td>4179</td>
<td>385</td>
<td>765</td>
<td>686.2</td>
</tr>
<tr>
<td>( \rho ) (kg/m³)</td>
<td>997.1</td>
<td>8933</td>
<td>3970</td>
<td>4250</td>
</tr>
<tr>
<td>( k ) (W/mK)</td>
<td>0.613</td>
<td>400</td>
<td>40.0</td>
<td>8.9538</td>
</tr>
<tr>
<td>( \alpha \times 10^7 ) (m²/s)</td>
<td>1.47</td>
<td>1163.1</td>
<td>131.7</td>
<td>30.7</td>
</tr>
<tr>
<td>( \beta \times 10^{-5} ) (1/K)</td>
<td>21.0</td>
<td>1.67</td>
<td>0.85</td>
<td>0.9</td>
</tr>
</tbody>
</table>

### NUMERICAL METHOD

The finite element method has been used to solve the nonlinear system of equations (11) to (14).

#### Variational Formulation

The variational form associated with equations (11) to (14) over a typical square element is given by:
\[
\int_{T} w_1 \left( U - \frac{\partial \psi}{\partial Y} \right) dX dY = 0.
\]

\[
\int_{T} w_2 \left( V - \frac{\partial \psi}{\partial X} \right) dX dY = 0.
\]

\[
\int_{T} w_3 \left( \frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} \right) dX dY = 0.
\]

#### Finite Element Formulation

The structure domain defined as: \( 0 \leq X \leq 1 \) and \( 0 \leq Y \leq 1 \) is discretized into square elements of same size. The finite element model, obtained from equations (22-26), by substituting finite element approximations of the form

\[
U = \sum_{j=1}^{4} U_j \xi_j, \quad V = \sum_{j=1}^{4} V_j \xi_j, \quad \psi = \sum_{j=1}^{4} \psi_j \xi_j,
\]

\[
\theta = \sum_{j=1}^{4} \theta_j \xi_j, \quad \Omega = \sum_{j=1}^{4} \Omega_j \xi_j,
\]

where \( w_1 = \xi_j \), \( w_2 = \xi_j \), \( w_3 = \xi_j \) and \( w_4 = \xi_j \) are arbitrary test function. All functions satisfy the homogeneous boundary conditions, as per theoretical requirements.
where \([k_m] \text{ and } [b^m] \) are defined as:

\[
K_y^{11} = \int \frac{\xi_y \xi_y}{\Omega} dX dY, \quad K_y^{12} = \frac{\partial^2 \xi_y}{\partial Y^2} dX dY, \quad K_y^{13} = K_y^{14} = K_y^{15} = 0
\]

(30)

\[
K_y^{22} = \int \frac{\xi_y \xi_y}{\Omega} dX dY, \quad K_y^{23} = -\int \frac{\partial^2 \xi_y}{\partial X \partial Y} dX dY, \quad K_y^{24} = K_y^{25} = 0
\]

(31)

\[
K_y^{33} = \frac{\partial^2 \xi_y}{\partial X^2} dX dY, \quad K_y^{34} = K_y^{35} = K_y^{45} = K_y^{55} = 0
\]

(32)

\[
K_y^{45} = \lambda \int \frac{\partial \xi_y}{\partial X} dX dY, \quad K_y^{41} = K_y^{42} = K_y^{43} = 0
\]

\[
K_y^{55} = \int \left( \frac{\partial^2 \xi_y}{\partial X^2} + \frac{\partial^2 \xi_y}{\partial Y^2} \right) dX dY
\]

(33)

The element equations have been solved by using Gauss elimination method by maintaining an accuracy of four significant digits.

### RESULTS AND DISCUSSIONS

The finite element method has been used to solve the nonlinear system of equations (11) to (14). The controlling parameter that define the fluid flow, heat and natural convection in an enclosure are aspect ratio \(A\), \(Gr\), \(\phi\) and \(Pr\). It needs quite extensive analysis to cover effects of each parameter. It is intended to limit the analysis for Grashof number \(10^3 \leq Gr \leq 10^5\), \(0 \leq \phi \leq 0.25\) as model of nanofluid and for an aspect ratio of 1 as a square enclosure. The thermophysical properties of fluid and solid phase are shown in table 1. A grid independence study is conducted using three different grid sizes of \(41 \times 41, 61 \times 61\) and \(81 \times 81\) for aspect ratio of 1. It is observed that further refinement of grids from \(61 \times 61\) to \(81 \times 81\) do not have a significant effect on the results.

Based upon these observations, a uniform grid of \(61 \times 61\) points is used for all calculations of aspect ratio of 1.

Fig. 1 (a)-(d) demonstrate the typical features of the volume fraction \(\phi\) on the stream line and isotherms for fixed value of the Grashof number. Fig. 1 (a) shows the representative sequence of stream line isotherms pattern in a square domain for \(\phi = 0\). In this study the flow consist of single roll, the tendency of which is to rearrange the fluid into a position of stable stratification, one in which the warm fluid that initially occupied the left half eventually moves to the upper half of the domain. Fig. 1 (b) to (d) indicates, as the volume fraction increases, the velocity at the centre of the enclosure increase as a result of higher solid-fluid transportation of heat. Flow strength also increases with increasing of volume fraction of nanoparticles.

The variation of the Nusselt number for different volume fraction of nanoparticle is shown in fig. 2. Volume fraction increases implies that more and more particles will be suspended so that thermal conductivity increased. Physically the heat transfer will also increase. It is clear from the graph that the heat transfer increases when increasing the volume fraction of nanoparticles.

The effect of Grashof number for pure fluid and nanofluids is depicted in Fig. 3. For a fixed aspect ratio, when Grashof number increases Nusselt number also increases. It is clear from the figure that the presence of the nanoparticles plays a significant role to increase the heat transfer.

Fig. 4 demonstrates the effect of particles diameter on Nusselt number. Comparison is also being done for Brinkman’s model and Pak and Cho’s Model for dynamic viscosity. Since four models have been taken for dynamic
viscosity but the results of Einstein’s model, Brinkman’s model and Brownian motion effects model are similar so only the comparison of two models have been taken. As the size of the nanoparticles increases, the Nusselt number of the nanofluid decreased. The ratio of the thermal conductivity is remarkably decreased as the size of the nanoparticles increases. So the Nusselt number of the nanofluids is decreased as the diameter of the nanoparticle increases.

Fig. 5 represents the variation of the Nusselt number with volume fraction using different nanoparticles for a fix value of Grashof number. The figure shows that the heat transfer increases almost monotonically with increase in volume fraction for all nanofluids.

CONCLUSION

The major findings contained in this paper are as follows: the natural convection of water based nanofluids is more stable than base fluids in a square enclosure heated from side, as the volume fraction of nanoparticles increases the size of the nanoparticle decreases, or the average temperature of nanofluids increases. In addition, as the viscosity increases the heat transfer coefficient derived from Brinkman’s model evaluating lower effective viscosity increase but the coefficient with Pak and Cho’s model is decreased. The results indicate that the Nusselt number increases as the volume fraction increases. The presence of the nanoparticles in the fluid changes the characteristics of the base fluids. A comparative study of different nanofluids based on the physical properties of nanoparticles is analyzed and found that Cu nanoparticles have high value of thermal diffusivity.

REFERENCES


Fig. 1 Stream line contours & Isotherms at various fraction parameters ($Gr = 10000, Pr = 0.7$)

Fig. 2 Variation of the Local Nusselt number along the hot wall for different Volume fraction ($Gr = 10000, Pr = 0.7$)

Fig. 3 Variation of the Local Nusselt number along the hot wall for different Grashoff number (Φ = 0.7, $Pr = 0.7$)

Fig. 4 Variation of the Local Nusselt number with particle diameter for different Model (Φ = 0.7, $Pr = 0.7$, $Gr = 10000$)

Fig. 5 Variation of the Local Nusselt number with Volume fraction for different nanofluids, ($Gr = 10000, Pr = 0.7$)