Ninth International Conference on CFD in the Minerals and Process Industries
CSIRO, Melbourne, Australia
10-12 December 2012

UPWARD JET PENETRATION IN FLUIDIZED BEDS : CFD PREDICTIONS COMPARED TO EXPERIMENTAL RESULTS

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ABSTRACT

Fluid catalytic cracking (FCC) process enables the production of Liquid Petroleum Gas, gasoline and heating oil by catalytic cracking of heavy hydrocarbons using several fluidized beds. In that context, gas jet penetration is an important parameter to consider in gas distributor design. It has a direct influence on the uniformity of the gas distribution and, therefore, on the process performances, internals erosion and catalyst attrition. The aim of this study is to evaluate the use of CFD to predict upward jet penetration.
Transient 3D simulations were performed using Barracuda CPFD. Earlier experiments performed on jet penetration at IFP Energies nouvelles (IFPEN) were simulated.
Corresponding results were then compared to experimental ones. The studied effects are the impacts of fluidization velocity, gas jet velocity, the initial bed height and the implemented drag law.
In a second step, three vertical jets were considered to see to what extent jets are interfering with one another. CFD results were analyzed thanks to the work of Merry (1971) and Hong and al. (2003) and a critical distance of interactions between jets was defined.
This study validates the use of CFD for upward jet penetration in the range of considered operating conditions.

NOMENCLATURE

C_D drag coefficient
d_p catalyst mean diameter
D coefficient
D_i i=1,2, coefficient
D_{gun} gun diameter
F_p particle drag force
H_{bed} initial bed heigth
H_j multijet penetration length
J jet penetration length
L_c critical distance between two jets
m_p particle mass
P_{atm} atmospheric pressure
P_P pressure
$P_{dynamic}$ dynamic pressure
P_{static} static pressure
Re Reynolds number
r_p particle radius
u_p particle velocity
u_f fluid velocity
$u_{fluidization}$ gas fluidization velocity
u_{gun} gas jet velocity at the gun outlet
u_{jet} gas jet velocity inside the jet

μ_f fluid viscosity
θ jet half angle
θ_p solid volume fraction
θ_{cp} close pack solid volume fraction
θ_f fluid volume fraction
ρ_p particle density
ρ_f fluid density
ρ_g gas density
ρ_{jet} jet density

INTRODUCTION

Jet penetration is an important parameter to consider in the gas distributors design. It has a direct influence on the uniformity of the gas distribution in the reactor and therefore on unit performances, internals erosion and catalyst attrition. As a consequence, a better knowledge of the length of penetration could allow a better optimization of the gas distributors design.
Many works have been conducted on jet penetration and are available in the literature. However, most of the time, those results can not be directly applicable. The first reason is the lack of exact definition of the length of penetration. Indeed, jet penetration length can be defined as many ways such as: the maximum or minimum observed jet length or the length obtained by measuring the momentum dissipation.
The second reason is that precision of experimental results is not always given. Previous studies at IFPEN showed that jet penetration length could not be given within a precision better than 20%. This information is really important as it can lead to a non-negligible overestimation of the jet penetration. A better accuracy in these results could imply a better control of the over-designs.
Based on former IFPEN experimental work, transient 3D CFD modelling was performed and the simulation results were compared to experimental results.
For the simulations, two drag laws were considered, as well as different operating conditions (jet velocity and initial bed height) and number of jets.
In the following sections, the considered methodology is presented and results concerning upward jet penetration are presented.

JET PENETRATION LENGTH DEFINITION

An exact definition of the length of penetration is not always given in the literature, especially with upward vertical jets. In that case, the length of penetration 'J' can be defined according to (see Figure 1):

- the maximum observed length
- the minimum observed length
- the length obtained by measuring the momentum dissipation.

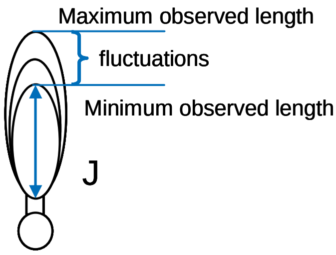

Figure 1: Length of penetration.

Moreover, the jet length oscillates a lot between two extreme positions (maximum and minimum observed lengths). These oscillations are called jet fluctuations.
Jet penetration has been widely investigated, experimentally or by CFD. We can classify the studied effect in three categories: geometry and number of guns, influence of gas operating conditions and influence of the type of solids.
Parameters such as the gun diameter or the type of solids are taken into account in the literature. For instance, Musmarra (2000) has studied several types of solids to establish an experimental database. On the other hand, as it has been reviewed by Wang et al. (2010) or Zhong and Zhang (2004), more than 20 different correlations describing the jet penetration length are avalaible and, as noted by Fiorentino and Newton (1998), some of them give totally different results.
The aim of CFD studies is not in most cases to obtain new correlations, but rather to validate a simulation model. Simulation results are always compared directly to experimental results or to existing correlations.
The main advantage of CFD is that it is possible to make easily vary geometric parameters such as the gun diameter. On the other hand, as Li et al. (2009) have remarked, authors are more confident in general patterns than in quantitative results. Zhong et al. (2007) have underlined the fact that 2D CFD simulations over-predicted the jet penetration length.
Thanks to former experimental studies, IFPEN has a strong basis of experimental data. That is why IFPEN has decided to validate CFD methodology using this database. Once this step will be done, it will be possible to have access to local information.

Definition of jet penetration considered for this CFD study

Jet penetration has been determined according to two methods. The first one, called 'visual' one, is based on the study of the solid density above the jet, after the steady state is reached. The length of penetration J follows the same definition as the experimental visual one, i.e. the distance from the tip of the nozzle to the length corresponding to the minimum of fluctuations of the first 'bubble' (dilute area). Then CFD simulation results can be directly compared to the experimental results.
Figure 2 gives an example of jet penetration definition in that case. It is assumed that the penetration length is given with the same precision as the experimental one, i.e. ± 20%.
The second definition, called 'momentum' one (similar to Pitot measurements), is based on the calculation of the relative jet momentum. It is defined as the ratio between the average observed momentum and the initial momentum at the nozzle outlet. Knowing the jet velocity, we can assume that, inside the jet, the mixture density is very close to the gas density.

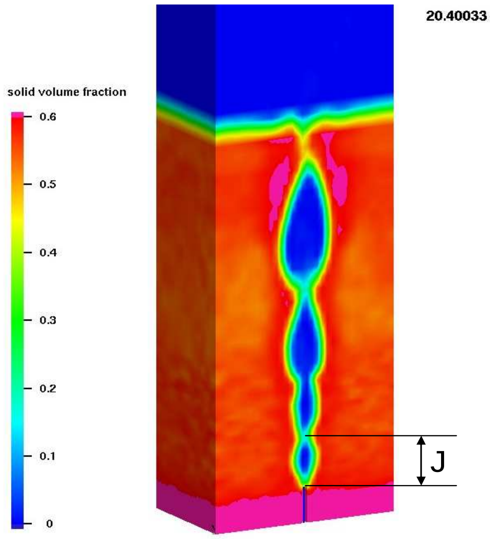

Figure 2: 'Visual' definition of jet penetration.

The relative momentum is defined as follows:

$$\Delta P_p = P_{dynamic} - P_{static} = \frac{1}{2}\rho_{jet} u_{jet}^2 \quad (1)$$

$$\Delta P_p^0 = \frac{1}{2}\rho_g u_{gun}^2 \quad (2)$$

$$relative\ momentum = \frac{\Delta P_p}{\Delta P_p^0} \quad (3)$$

where ρ_{jet} is taken equal to ρ_{gun}.

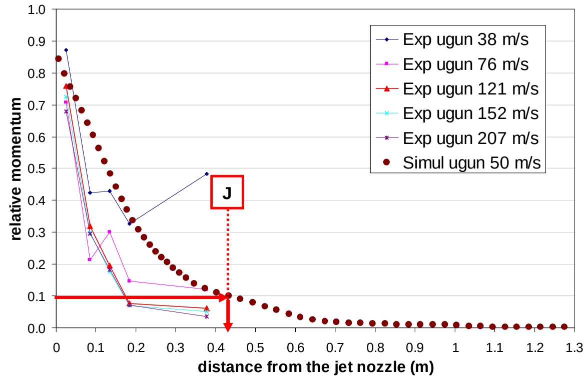

Figure 3: Evolution of relative momentum versus the distance from the jet – CFD (run3 conditions) compared to experimental results.

As shown on Figure 3, the relative momentum dissipates a lot initially and then tends to reach a plateau: initially, the jet is almost fully gas, so the relative dissipation represents a progressive decrease of the gas velocity. Then, reaching the plateau, particles start to incorporate the jet. The transition between the two zones defines jet penetration.
Only the vertical contribution of the jet velocity is integrated in the calculation of the simulated momentum. If this momentum is compared to former experimental results (obtained for a fluidization velocity of 1cm/s), simulated jet penetration length is slightly over predicted but the overall shape of the curve is similar.

By definition, jet penetration length is obtained for a decrease of 90% of the relative momentum.

IFPEN EXPERIMENTAL STUDY

In the late 90's, IFPEN conducted experiments in order to improve jet penetration understanding.
The experimental arrangement is shown in Figure 4. The main characteristics of the unit are reported in Table 1.

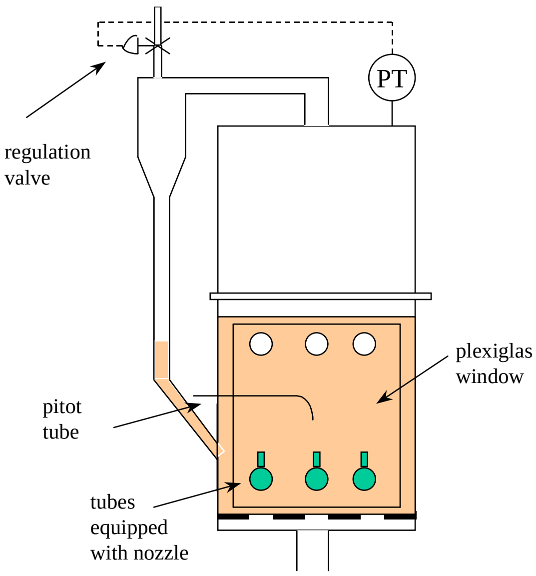

Figure 4: IFPEN experimental set-up.

size	600 x 500 x 3100 mm^3
solid inventory	300 kg
solid properties	FCC catalyst – 1400 kg/m^3 – d_p = 70µm
bed level	1.3 m
relative pressure in the vessel	0 to 1.5 barg - tests made at 0 barg
fluidization velocity	up to 0.3 m/s at 1.5 barg - tests at 0.5 cm/s at 0 barg
number of tubes for nozzle testing	3 (but only 1 at the same time for most of the tests)
nozzle outlet diameter	14 or 30 mm
maximum nozzle outlet velocity	100 m/s for larger nozzles (30 mm diameter)
nozzle outlet velocity	50/80/100 m/s at 0 barg
nozzle orientation	Vertical Upwards, Vertical Downwards, Horizontal, 45° downwards

Table 1: IFPEN experimental set-up.

However, it was difficult, especially for vertical jets, to obtain reliable results only with visual observation.
First tests were performed visualizing the jet through a window pane. The distance between the nozzle and the window pane was reduced to 3 mm. Images were recorded with a video camera.
Jet penetration length was also deduced from the evaluation of the jet momentum by inserting a Pitot tube at defined positions in several parts of the jet. Air back-flush was used to avoid plugging of the nozzle.
The two methods gave consistent results.

CFD STUDY

CFD Transient 3D simulations were performed using. Barracuda CPFD software. The governing equations of the fluid phase are solved using a continuum model and those of the particle phase using a Lagrangian model.
At given operating conditions, two drag laws have been successively tested : Wen and Yu and Wen-Yu-Ergun.
Finally, CFD results have been compared to experimental results.

Operating conditions and simulated geometry

The simulated geometry is presented on Figure 5. This configuration is based on IFPEN experimental set-up. The outlet is set at atmospheric pressure (in yellow). Fluidization air is injected at the bottom (in red). Boundary conditions for the jets are located at the top of each jet (in red). Gravity is constant in every simulation and is vertical downward.

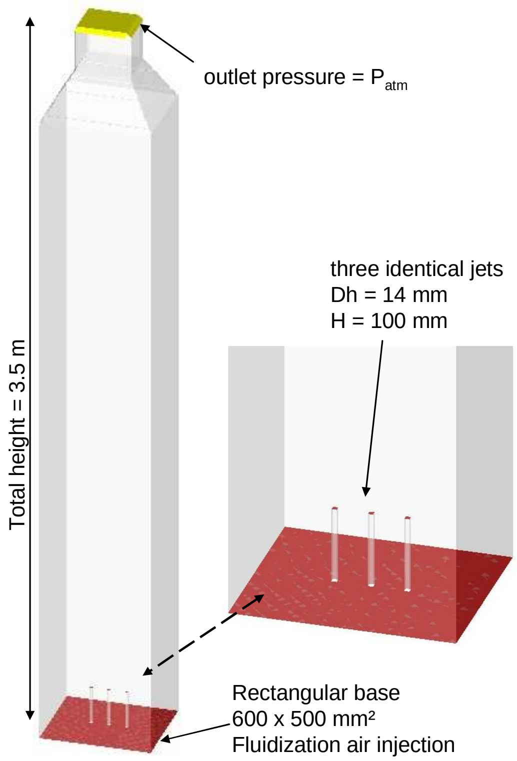

Figure 5: Simulated geometry : in the case of only one vertical jet, only the central one is working (in yellow : pressure boundary conditions, in red : flow boundary conditions)

In a second step, three vertical upward jets have been considered. The distance between the jets is 95 mm. In the past, experiments with jets distant from 190 mm performed at IFPEN showed no interactions between jets, whatever the gas velocity (50, 80 or 100 m/s).
The inter-jet distance has been reduced by half to estimate the extent of inter-jet interactions, depending on the jet velocity.

Table 2 gives the simulated geometry dimensions and the operating conditions.

dimensions	600 x 500 x 3500 mm^3
initial bed level	1, 1.3 or 2m
solid properties	FCC catalyst – 1400kg/m^3 – d_p = 70µm
relative pressure in the vessel	0 barg
fluidization velocity	0 or 5mm/s
number of tubes for nozzle testing	1 and 3 distant from 95mm
nozzles outlet hydraulic diameter	14 mm
nozzle outlet velocity	5, 50, 80, 100 or 110m/s
nozzle orientation	Vertical Upwards

Table 2: IFPEN simulation set-up.

Simulation parameters

The fluid drag on a particle is described by the equation (Wen and Yu, 1966) :

$$\vec{F}_p = m_p D(\vec{u}_f - \vec{u}_p) \quad (4)$$

D is linked to the drag coefficient via the following relation:

$$D = 0.375 C_D \frac{\rho_f |\vec{u}_f - \vec{u}_p|}{\rho_p r_p} \quad (5)$$

Depending on the Reynolds number, C_D will take the following values:

Re < 0.5 $$C_D = \frac{24}{\text{Re}} \theta_f^{-2.65} \quad (6)$$

$0.5 \le \text{Re} \le 1{,}000$

$$C_D = \frac{24}{\text{Re}} \theta_f^{-2.65} \left(1 + 0.15 \,\text{Re}^{0.687}\right) \quad (7)$$

Re > 1000 $$C_D = 0.44 \theta_f^{-2.65} \quad (8)$$

with $\text{Re} = 2 \frac{r_p \rho_f |\vec{u}_f - \vec{u}_p|}{\mu_f}$.

To test the influence of drag law, Wen-Yu-Ergun model was considered.

Wen-Yu-Ergun drag law is based on Wen and Yu model but drag force is represented by the equation :

$$\vec{F}_p = m_p D_i (\vec{u}_f - \vec{u}_p) \text{, i=1 or 2} \quad (9)$$

with $$D_1 = 0.375 C_D \frac{\rho_f |\vec{u}_f - \vec{u}_p|}{\rho_p r_p} \quad (10)$$

C_D being calculated using Wen and Yu equations.

$$D_2 = 0.5 \left(\frac{C_1 \theta_p}{\theta_f \,\text{Re}} + C_2 \right) \frac{\rho_f |\vec{u}_f - \vec{u}_p|}{\rho_p r_p} \text{ (Ergun)} \quad (11)$$

D_1 and D_2 are used, depending on the solid volume fraction value:

$\theta_p > 0.85\theta_{CP}$ D=D_2 (12)

$\theta_p < 0.75\theta_{CP}$ D=D_1 (13)

$0.85\theta_{CP} \ge \theta_P \ge 0.75\theta_{CP}$

$$D = \frac{\theta_P - 0.85\theta_{CP}}{0.85\theta_{CP} - 0.75\theta_{CP}} (D_2 - D_1) + D_1 \quad (14)$$

In this study, the impacts of fluidization velocity, gas jet velocity, initial bed height and number of jets on jet penetration length are presented (see Table 3).

For all the cases, a complete solids distribution has been considered and not only a median diameter (70µm). As a consequence, this simulation is supposed to well represent reality in terms of particles interactions and entrainment.

Except for run8 cases where the bed is not fluidized, all the other cases have been run with a fluidization velocity of 5 mm/s. This value leads to a mean bed density of 0.5.

	Drag law	u_{gun} (m/s)	H_{bed} (m)	simulated time (s)
run3	Wen and Yu	50	1	30
run4	Wen and Yu	50	2	46
run6-1	Wen and Yu	50	1.3	30
run6-2	Wen and Yu	80	1.3	30
run6-3	Wen and Yu	110	1.3	30
run8-1	Wen and Yu	50	1.3	30
run8-2	Wen and Yu	80	1.3	15
run8-3	Wen and Yu	110	1.3	15
run10-1	Wen and Yu + Ergun	50	1.3	15
run10-2		80	1.3	15
run12-1	Wen and Yu + Ergun	50	1.3	15
run12-2		80	1.3	15
run12-3		100	1.3	15

Table 3: Simulation cases.

In the following sections, jet penetration length is defined in the case of CFD studies. It is then compared to jet penetration length based on experimental results

RESULTS

Vertical upward single jet penetration results

Simulation results are summarized in Table 4 and compared to experimental jet penetration length.

	experimental	CFD J(m) visual	CFD J(m) momentum
run3	0.18	0.17	0.43
run4	0.18	0.18	0.16
run6-1	0.18	0.18	0.30
run6-2	0.22	0.20	0.37
run6-3	0.24	0.23	0.36
run8-1	0.18	0.19	0.29
run8-2	0.22	0.19	0.31
run8-3	0.24	0.21	0.33
run10-1	0.18	0.15	0.31
run10-2	0.22	0.22	0.40

Table 4: Jet penetration results: comparison with IFP Energies experimental results.

Jet Penetration length definition for CFD

In this section, results of jet penetration length determined by visual and momentum methods for CFD are discussed.

As it was the case experimentally with Pitot measurements, jet penetration estimated thanks to the momentum determination is overestimated compared to visual determination. However, in our case, the difference between the two methods is quite high, especially for low velocities (see results in Table 4).

The first explanation is linked to the fact that only the vertical contribution of the jet velocity is integrated in the

calculation. Maybe the influence of the other contributions is more important than expected for low jet velocities.
The second explanation is linked to the use of Wen and Yu drag law (or its contribution in Wen-Yu-Ergun drag law). In fact, this law is usually over predicting the presence of the dilute phase: the central jet velocity is then overestimated, and so the calculated jet penetration.
This effect is not so obvious in the case of the visual prediction as the whole jet shape is taken into account.
For those reasons, for CFD results, we consider jet penetration length only by the visual method. However, this methodology is not really practical.

Drag law effect

Considering Wen and Yu drag law (run6 cases) or Wen-Yu-Ergun drag law (run10 cases), both simulations results are in the same range of the experimental results (see Figure 6).

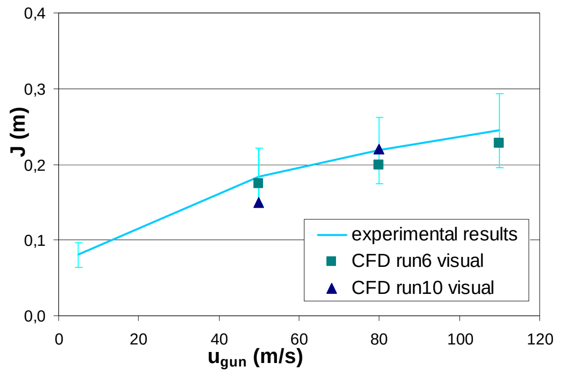

Figure 6: Influence of jet velocity and implemented drag law on jet penetration length.

Wen and Yu drag law and Wen-Yu-Ergun drag law are therefore equivalent. In fact, in the latter, drag law is the sum of two contributions that are weighted depending on the bed density: Wen and Yu contribution is predominant in the central part of the jet (low bed density) compared to the Ergun part (predominant far from the jet).

In the rest of the study, the different effects are investigated considering Wen and Yu drag law.

Effect of jet velocity and fluidization velocity

Jet velocity values have been varied from 5 to 110m/s, going through 50 and 80m/s. Results obtained in the case of run6 and run8 are presented on Figure 7.

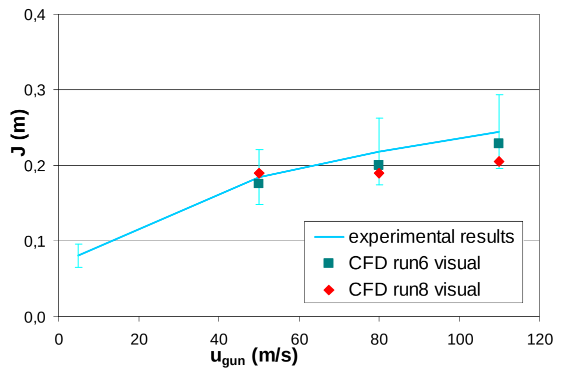

Figure 7: Influence of jet velocity and fluidization velocity on jet penetration length.

CFD jet penetration length as a function of the jet velocity is in very good agreement with the one observed experimentally.
Gas fluidization velocity does not seem to have a real effect on the visual penetration as the fluidization velocity was very low (0 compared to 5mm/s) in the simulations: bed density goes from 698 to 700kg/m^3.
As a consequence, visual jet penetration does not seem to depend on the gas fluidization velocity for very small values of $u_{fluidization}$. However, former experimentations showed that the jet shape is different depending upon whether the bed is fluidized or not.

Effect of initial bed height

The influence of the initial bed height is presented on Figure 8.
Initial bed height has a negligible effect influence on jet penetration length.

This result consolidates former experimental studies where no influence of the initial bed height is taken into account.

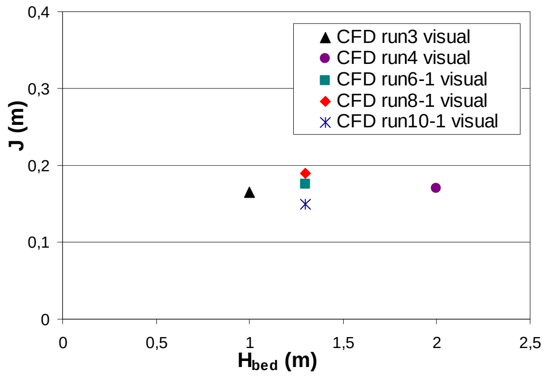

Figure 8: Influence of initial bed height on jet penetration length (for a jet velocity of 50m/s).

This result may seem counter-intuitive as a greater initial bed height may lead to greater compaction. It should be then more difficult for the jet to penetrate. In fact chosen initial bed heights may be not great enough to show this possible phenomenon.

Effect of the number of guns : interactions between jets

Three vertical upward jets have been considered. The inter-jet distance was 95 mm. In earlier experiments, for an inter-jet distance of 190 mm, no significant interaction was detected. In this study, this distance has been halved, in order to estimate the interaction as a function of jet velocity.

Interactions between jets can affect the prediction of the jet penetration length. Merry (1971) has investigated the influence of jet velocity, nozzle width and particle properties on the jet penetration length using fluidized beds with different kinds of particles. He developed a simple model for calculating the jet angle by correlating experimental data:

$$\tan\theta = \frac{1}{10.4}\left(\frac{\rho_P d_p}{\rho_g D_{gun}}\right) \tag{15}$$

The critical distance between two jets, Lc, can be calculated using the following equation:

$$L_c = D_{gun} + 2.H_j.\tan\theta \tag{16}$$

For Hong and al. (2003), H_j stands for the jet penetration length in the case of multijets.

θ is defined on Figure 9:

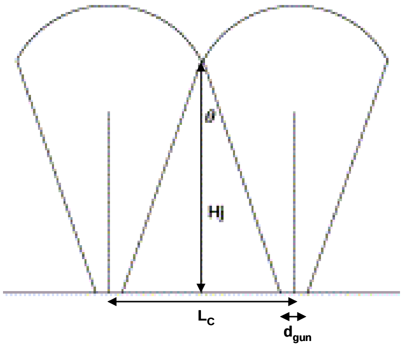

Figure 9 : Interaction between two jets: definition of the jet half angle

On the other hand, in the present study, if we try to calculate the multijets penetration length, H_j, it will be constant and equal to 25 cm. This value is not depending on the jet velocity. As a consequence, the calculation of the jet penetration length using equation (16) cannot be used as a prediction tool concerning jet interaction. It can however be used to calculate the critical distance between jets for which interaction may occur.

If we consider the results of Table 5 in a predictive mode, i.e. using jet penetration obtained in the case of a single jet, we remark that the critical distance between the jets is below the real distance (95 mm) for the first case (u_{gun}=50m/s) and has the same order of magnitude for the last one (u_{gun}= 100m/s). That means that jets should not interact for the first case while should interact for the last case.

u_{gun} (m/s)	Lc (mm) using J(m) – one jet experimental	Lc (mm) using J(m) – one jet 'visual' CFD
50	74	72
80	85	79
100	91	87

Table 5: Jet penetration results: calculation of the critical distance between jets

Table 6 presents jet penetration obtained experimentally and by CFD for one jet and by CFD for the three jets considering visual method.

u_{gun} (m/s)	one jet J(m) experimental	one jet J(m) CFD visual	CFD J(m) visual
50	0.18	0.18	0.12
80	0.22	0.20	0.18
100	0.24	0.23	0.18

Table 6: Jet penetration results: comparison between one and three jets

Case with no jet interaction:

For the lower jet velocity, as expected, jet penetration length for each is not impacted by the presence of the two others and CFD results are similar to the one obtained in the case of a single jet.

As presented on Figure 10 jets interaction occurs higher in the bed and obviously due to the oscillation of the jet along the bed.

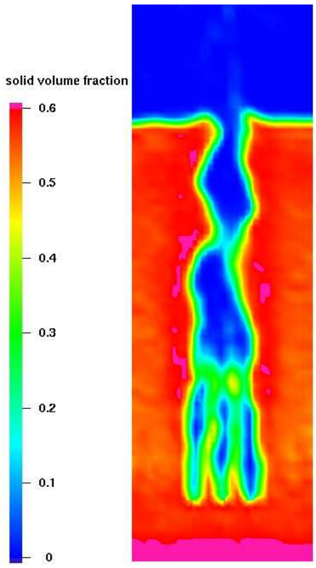

Figure 10 : Interaction between jets (u_{gun} = 50m/s)

Cases with jet interaction:

For the two other cases, jet penetration length is slightly reduced due to jet interactions. Jets are merging and the part of the jet below this merging is lower than the penetration length in the case of an individual jet.

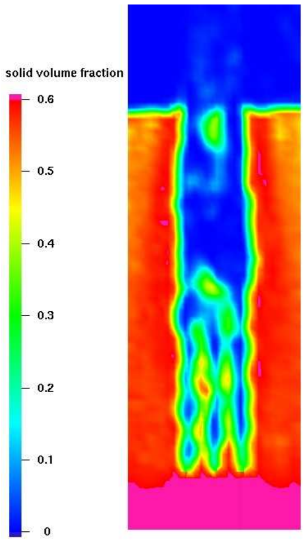

Figure 11 : Interaction between jets (u_{gun} = 80m/s)

In fact, as shown in Figure 11, the jets tend to merge into a single wider jet.
Jets are interacting also with the fluidized bed. A lot of particles, between the jets as long as besides the jets, are entrained in the jet.

The critical distance between jets can be however evaluated thanks to equation (16) keeping into account a safety margin of the same order of magnitude as the one of the jet penetration length in the case of only one jet.

CONCLUSIONS

Two methods were used to define simulated jet penetration length: the first one is based on the visual description of the jet and the second one is based on the momentum calculation; 'visual' jet penetration is a better method to describe CFD results. However, this method is not very practical.

CFD simulations, on the base of Wen and Yu or Wen-Yu-Ergun **drag law**, gave a **good approximation** of IFPEN **experimental results**, keeping in mind that experimental results are given within ±20%. Both drag laws considered for this study can be used to represent vertical upward jet penetration.

Effects of **gas jet velocity** as well as **gas fluidization velocity** are in agreement with experiments. However, as the fluidization velocity was very low (5mm/s), it may be then interesting to test a higher fluidization gas velocity to see to what extend this parameter influence the jet penetration. On the other hand, jet shape should be defined to see to what extent gas fluidization has an effect.

Initial bed height has not a real influence on jet penetration length. This result **consolidates former experimental studies** where no influence of the initial bed height was noticed.

In the second part of this study, thanks to the work of Merry (1971) and Hong and al. (2003), a **critical distance** of **interaction between jets** has been calculated. This distance has to take into account in its calculation a safety margin of the same order of magnitude as the one of the jet penetration length in the case of only one jet. For a given jet velocity, if the jets are located further this critical distance, jet penetration length is not impacted. If the jet distance is lower, jets penetration length will be reduced. However, in all the cases, jets are **merging further** in the bed and are entraining a lot of particles.

This work has been carried out within the FCC Alliance program (developed by IFPEN, Axens, Shaw and Total).

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