Convection Due to Horizontal Shaking

Kurt Liffman[1], Guy Metcalfe[1], and P. Cleary[2]

1. Advanced Fluid Dynamics Laboratory, CSIRO/DBCE, Highett, 3190, Australia
2. CSIRO/DMIS, Private Bag 10, Rosebank MDC, 3169, Australia

ABSTRACT

We use a discrete element code to vibrate a monodisperse particle bed in the horizontal direction. We obtain a convective surface layer extending into the particle bed some 10 particle diameters from the surface. This surface convective motion can give rise to a "reverse Muesli effect", i.e. where the large particles, initially, move towards the bottom of the particle bed.

For particle beds with heights greater than 10 particle diameters, and for $\Gamma \gtrsim 1.2$, one obtains interior, counter-convective rolls, where the material travels down the vertical side walls and up the center of the pile. This latter result is consistent with experimental observations.

1. INTRODUCTION

Size segregation often occurs when a collection of differently sized particles is subject to vibrational motion. One common property of this size segregation is the, so-called, "Muesli effect", where if one shakes a packet of muesli, the larger particles tend to rise to the top of the pile. Such behavior appears to be driven by vibration-induced convection (Jaeger *et al.* 1994), where the convection is dependent on the friction between the particles and the vertical side walls of a container. "Low" friction material does not move or convects up the side walls, while "high" friction material convects down the sidewalls (Behringer 1993, Herrmann 1993).

A relatively unexplored question relates to the behavior of particles that are subjected to horizontal shaking. To research this problem, we have developed a hybrid particle code, which we will now describe.

2. MODEL

The hybrid code combines a hard sphere model (Campbell & Brennan 1985) with a particle dynamics method (Cundall & Stack 1979). Such hybrid models typically have two particle-collision regimes: "hard" and "soft".

In a hard collision, the post-collisional velocities are computed from momentum conservation equations. The resulting momentum transfer equations are readily available from the literature (*e.g.* Hopkins 1987, Liffman *et al.* 1992), where energy loss is modelled with a coefficient of restitution (e_r) and a slip-friction coefficient (μ_{slip}). In the simulations shown in this paper, we have set $e_r = 0.7$ and $\mu_{slip} = 0.5$

In our code, hard collisions occur for particles that have "large" relative velocities. If the relative velocities become "small", then the particles can inter-penetrate. The particle-particle interactions can then be represented by a damped spring acting along the line joining the centers of the two particles (Cundall & Stack 1979). The equation of motion for this interaction is simply the damped harmonic oscillator

$$m\Delta\ddot{\mathbf{r}} + c\Delta\dot{\mathbf{r}} + k\Delta\mathbf{r} = -m\mathbf{g} + \mathbf{R} \ , \qquad (1)$$

where m is the mass of the particle, $\Delta\mathbf{r}$ is the interpenetration distance, c the damping factor, k the spring constant, $\mathbf{g}$ gravity, and $\mathbf{R}$ is the summation of other body forces. The coefficient of restitution for a soft collision is obtained from the homogeneous solution of Eq. (1) and has the form

$$e_r = \exp\left(-\frac{\pi c}{\sqrt{4mk - c^2}}\right) \ . \qquad (2)$$

By using Eq. (2), we can adjust k and c so that e_r for both soft and hard collisions is the same.

As is true for all current discrete-element codes (Herrmann 1993), static friction is not included for soft interactions between the particles. However, for interactions between the particles and the walls, the code simulates static friction, slip friction and rolling friction (Witters & Duymelinck 1985), where the coefficient of rolling friction (μ_{roll}) is set to the value 0.01.

3. CONVECTION AND HORIZONTAL SHAKING

To examine, computationally, the flow regimes that occur when a collection of monodisperse particles is subject to horizontal shaking, we consider an initial particle configuration, where 900 disks have been placed into a 3 cm × 9 cm box. Each disk has a diameter of 1 mm and a mass density of 3 gm cm^{-3}.

We shake the box in the $\hat{x}$ direction, where the amplitude of the shaking (x) as a function of time (t) is given by

$$x(t) = A \sin(2\pi t/T_p) \tag{3}$$

where A is the amplitude of the motion, and T_p is the period of oscillation.

We set $A = 0.5$ cm, and $T_p = 0.1$ s. The value of $\Gamma = A\omega^2/g$, ($\omega = 2\pi/T_p$) in this case is 2.01. As illustrated by the displacement vectors in Fig. 1, horizontal sinusoidal shaking produces four separate convection cells. The two upper convection rolls flow downward at the center of the box and flow upwards as one moves closer to the vertical walls of the box. The two lower convection rolls flow up at the center of the box and down the vertical walls of the box. The lower convection rolls appear to be consistent with experimental results (Evesque 1992).

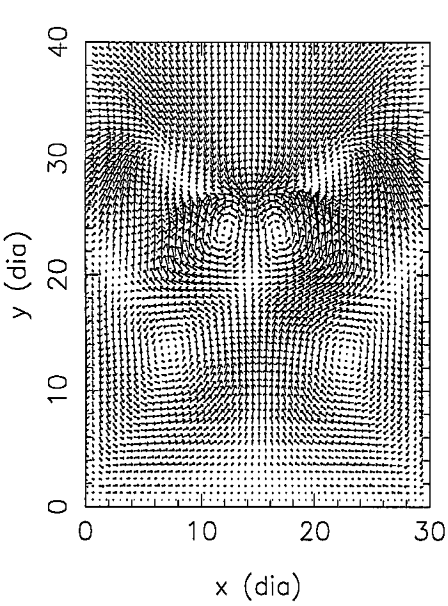

Figure 1: Particle displacement vectors obtained from horizontal sinusoidal shaking for $\Gamma = 2.01$, where distances are measured in units of particle diameters

An examination of the top half of Fig. 1 reveals displacement vectors that appear to defy continuity, *i.e.*, vectors that appear to be a source of particles, when - by construction - no such particle source exists. This effect arises because Fig. 1 shows the "phase-averaged" displacements of the particles, *i.e.*, we sample the positions of the particles at one point in the vibration cycle and then sample the subsequent position of the particle, one cycle period, T_p, later. Particles that move on small, roughly, linear paths in the cycle period will satisfy continuity. However, particles that move on relatively large, looped paths may not appear to satisfy continuity.

The lower convection rolls, shown in Fig. 1. convect slowly relative to T_p, and so preserve continuity in our phase-averaged plot. To illustrate the onset of convection, we plot, in Fig. 2, $\mathcal{J}$ vs Γ, where $\mathcal{J}$ is a measure of the total displacement (measured in particle diameters) suffered by the particles in the region of the pile where the lower convection rolls take place.

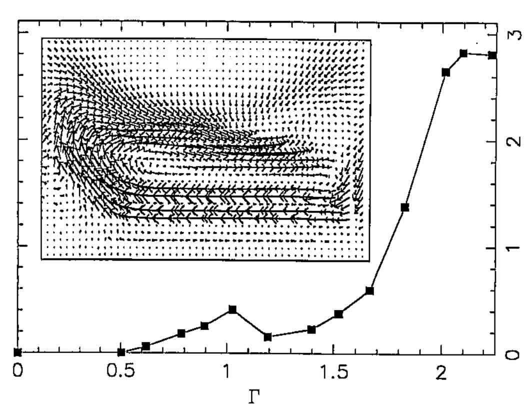

Figure 2: The average displacement per cell $\mathcal{J}$ as a function of Γ. Below $\Gamma = 1$, there is one convection cell (see inset for $\Gamma = 0.895$ flow), while for $\Gamma > 1.2$ the two interior convection cells begin to appear.

The lower convective rolls are not observed for $\Gamma \leq 1$. Instead, as is shown in the inset of Fig. 2, a single convection roll is observed, where the particles travel, unidirectionally, down the free surface and cycle back through the interior of the heap.

The onset of the convective rolls occurs for $\Gamma \gtrsim 1.2$ - a result that is similar to convection induced by vertical shaking Taguchi (1992). The flattening of $\mathcal{J}$ for $\Gamma \gtrsim 2$ is a sampling effect due to a change in the shape of the pile. For $\Gamma \gtrsim 2$ the pile is periodically squashed against one wall and then the other. This causes the heap to be distended in the y direction, so the top of the convection rolls exceeds our sampling region and thereby causes a decrease in $\mathcal{J}$.

The mechanism that produces these convection rolls is related to, but somewhat different from, what is thought to occur for the vertical-shaking case.

Convection due to vertical shaking is, in part, produced by the friction between the particles and the side walls. For horizontal-shaking, however, the convection is produced from a simple gravity effect. As the box moves from side to side, an intermittent gap is created between the particle heap and the side walls. Such a space allows surface particles to simply roll/slide down the side of the heap. When the gap closes, these "new" particles are pushed into the interior of the heap, thereby driving the flow.

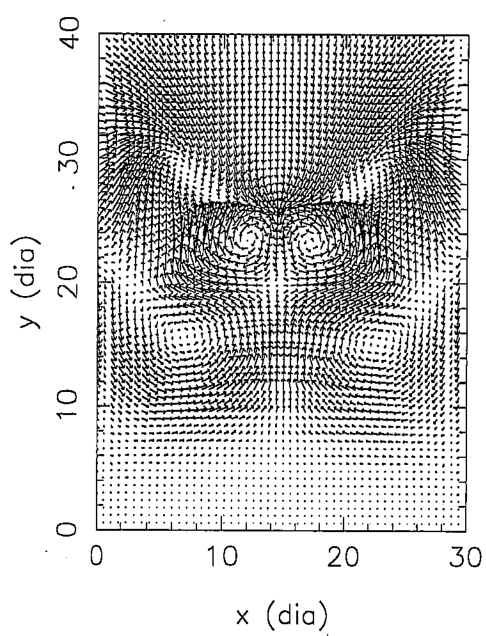

Figure 3: Particle displacement vectors obtained from horizontal sinusoidal shaking for $\Gamma = 1.83$.

From such a simple model, one can predict that the lowest point in the convection cells should be proportional to the "separation point" of the heap, *i.e.*, the height at which the heap separates from the wall. At distances lower than the separation point, the heap is still, effectively, touching the wall, so particles cannot roll or slip further than this point.

The separation point decreases in height as Γ increases. So, one should expect that the lowest point of the convection roll should move lower with increasing Γ, and indeed, as Figs 1 and 3 illustrate, this is what is observed. This effect is shown, more quantitatively, in Fig. 4, where we plot the separation point (y_s) and the lowest point of the convection rolls (y_c) as a function of Γ. There is a good correlation between the convection and separation points, which is consistent with our simple model for the formation of the convection.

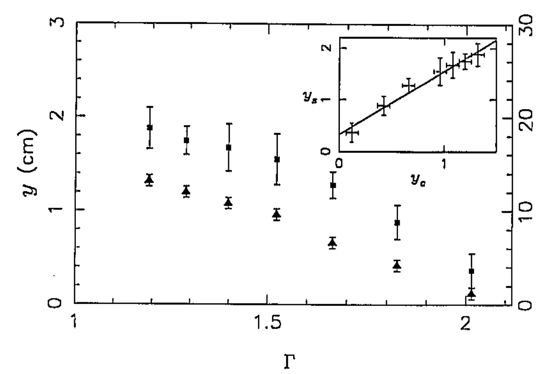

Figure 4: Separation points (squares) and convection points (triangles) as a function of Γ. The y values for these points are scaled in cm and particle diameters. The inset graph illustrates the correlation for the separation point distance (y_s) as a function of the convection point distance (y_c). Note, that in this graph, the separation point is defined to the value of y where the heap is one particle diameter away from the side wall. Below this point, the gap between the heap and the side wall is less than one diameter.

4. THE REVERSE MUESLI EFFECT

If we place a "large" (2 mm diameter, 3 g cm^{-3}) particle at the top center of the heap and apply a $\Gamma = 2.0$ vibration, the particle will trace out the top surface-wave convection rolls shown in Fig. 1, *i.e.*, it will move swiftly downwards until it reaches the $y \approx 20$ diameter point and then move sideways and up, to start the cycle anew. A decrease in height of the pile will cause a decrease in the height of the lower convection rolls, but the behavior of the top surface-convection rolls will remain relatively unchanged.

Indeed, if a pile is 1.0 cm or 10 particle diameters high, the large particle will eventually reach the ground and remain there for some time. This gives rise to a "reverse-muesli" effect, where the particle moves in the opposite direction to what is, typically, observed. In Fig. 5 we display the phase-averaged convection pattern for a heap that is 10 particle diameters high. The reverse-muesli effect is an obvious consequence of this convection pattern.

In Fig. 6, we display the height (y value) of the large particle as a function of time. We consider two examples of horizontal shaking: a 15 dia and 30 dia heap, plus another example of a ~ 30 dia heap, except now the box is subject to a $\Gamma = 2.01$ ($A = 0.5$ cm, $T_p = 0.1$) sinusoidal vibration in the vertical direction.

As can be seen, the particle that is subject to vertical vibration moves upwards, while the particles subject to horizontal vibration move downwards. For the case of horizontal shaking, the large particle is able to reach the ground only when it is placed on the smaller pile, thereby reflecting the occurrence of the lower convection rolls in the larger pile.

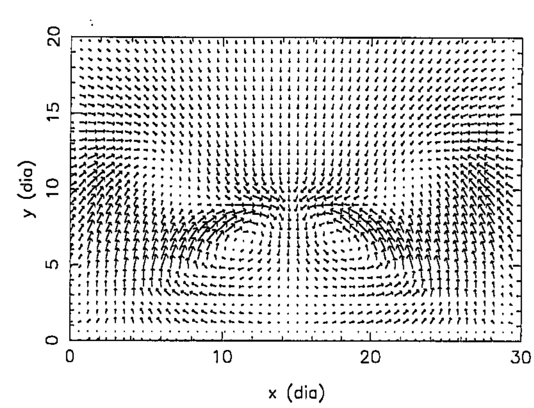

Figure 5: Particle displacement vectors obtained from horizontal, sinusoidal shaking ($\Gamma = 2.01$) of a 10 particle diameter high heap.

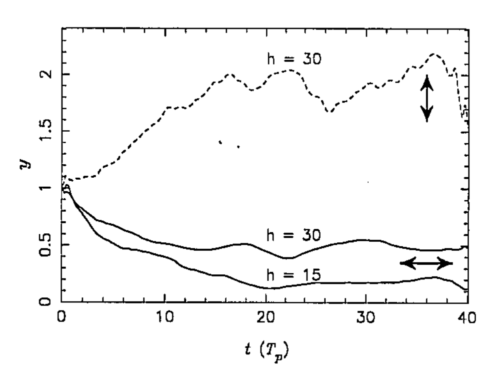

Figure 6: The height of a large particle (y) as a function of time (t) for horizontal and vertical shaking. Time is displayed in units of the shaking period (T_p), while the height is normalized to the original height of the pile (h). The two solid lines show the downward movement of the large particle when the piles are subject to horizontal ($\Gamma = 2.0$) shaking (represented schematically by the horizontal arrows). The two h values give the height of the pile in particle diameters. The upper dashed line shows the upward motion of the large particle when the pile is subject to vertical ($\Gamma = 2.0$) shaking. Note that, for illustrative purposes, we have translated this line upward by one unit.

CONCLUSIONS

We have demonstrated, using a discrete element code, that horizontal shaking can produce two sets of convection cells in a box of granular material. The upper set of convection cells arises due to surface-wave effects, with material convecting up the side walls and down at the center of the heap. The lower convection cells move in the opposite direction, *i.e.*, down the side walls and up the center of the heap.

The behavior of these lower convection cells is consistent with qualitative experimental observations. They arise due to the, gravity-induced, downward movement of particles in the intermittent gaps that form between the heap and the sidewalls.

This convective behavior gives rise to a "reverse-muesli" effect, where a large particle placed at the top-center of the pile will convect downwards until it reaches the upwelling of the lower convection rolls. For "small" heaps, the large particle will actually reach the ground. In both cases, the surface-wave convection may cause a "large" particle to return to the top of the pile

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