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Modelling Gas Injection of a Peirce-Smith Converter

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ABSTRACT

A commercial CFD-code PHOENICS was used to solve isothermal flow field of gas and liquid in a Peirce-Smith converter. An Euler-Euler based algorithm was chosen for modelling fluid dynamics and evaluating controlling forces of a submerged gas injection. Predictions were made with a k-ε turbulence model in the body fitted co-ordinate system.

The model has been verified with a ¼ scale water model, and a parametric study with the mathematical model of submerged gas injection was made for the PS-process and the ladle injection processes. Limits of the modelling technique used were recognised, but calculated results indicates that the present model predicts the general flow field with reasonable accuracy. Predicted bubble distribution, pattern of the flow field and magnitude of flow velocities were used to evaluate scaling factors of physical models and general flow conditions of an industrial PS-converter.

NOMENCLATURE

d_n	diameter of nozzle or tuyere (m)
d_v	diameter of a cylindrical vessel (m)
g	gravitational acceleration (m/s^2)
h_b	height of liquid bath (m)
h_n	height of free surface from the nozzle or tuyere (submergence) (m)
Q	gas flow rate (m^3/s)
t	time (s)
V	volume of a bubble (m^3)
μ	dynamic viscosity (kg/ms)

1. INTRODUCTION

Even though the Peirce-Smith converter has been the major blister copper production route for decades, there still remain some unsolved problems, which lower considerably the productivity of the PS-process. The most serious problems are tuyere blockage and severe refractory erosion. Continuous punching of the tuyeres is required to remove accretions that block the gas flow, and refractory wear at the tuyere line is usually the limiting factor in the campaign life of a converter. These defects are accepted as an inevitable part of the process, but efforts to overcome or reduce them have been continuous. As a result of the general research done on the PS-process, Brimacombe et al., 1984 have suggested that the tuyere blockage could be avoided and Kimura et al., 1986 have suggested that the refractory wear could be significantly reduced by high-pressure gas injection. However, economical advantages were found to be sensitive to the capital and power costs of raising the blowing pressure (Garrido and Lee, 1987), and high-pressure injection has not become a common practice in copper converters. The large number of unknown factors in evaluating the process development investments is probably the main reason why the Peirce-Smith converter has not undergone any major changes during the past decades. The aim of this study is to use fluid flow simulations to be able to predict better the consequences of modifications made to the PS-converter operation.

2. METHOD OF ANALYSIS

2.1 Previous Work and Theoretical Considerations

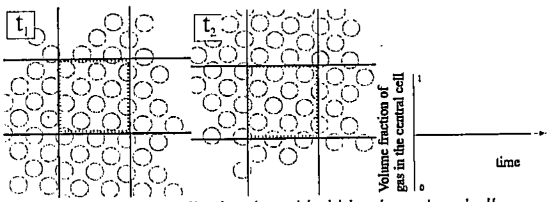
Because submerged gas injection is a common application in the metallurgical industry, it have been studied intensively and a great deal of physical and mathematical models of the injection processes has been published. The mathematical modelling studies are mainly concentrated in the field of ladle metallurgy. Many recent publications on progress made in the fluid flow simulations of ladles have been issued by e.g. Türkoglu and Farouk, 1991, Ilegbusi and Szekely, 1990, Sheng and Irons, 1995, Mazumdar and Guthrie, 1995, Schwarz, 1996, and Jönsson and Jonsson, 1996. There is also a large number of published experimental measurements made with laboratory scale water models of ladles e.g. Castillejos and Brimacombe, 1987, Johansen et al., 1988, Sheng and Irons, 1992, and Iguchi et al. 1995. These comprehensive experimental data on fluid flows of the ladle water models provide a feasible basis for the verification of the mathematical models.

Even though physical models have also been made for PS-converters, quantitative data on flow dynamics is limited. Most modelling studies have concentrated on particular phenomena like tuyere blockage, bubble formation, mass transfer, slopping and high pressure injection. Physical modelling, as well as pilot and industrial scale experiments, of the Peirce Smith-converter have published by e.g. Bustos et al., 1894, Adjei and Richards, 1991, Liow and Gray, 1990, Kimura et al., 1986 and Brimacombe et al., 1985. Although some of the modelling studies have focused on the flow velocities (Hein and Schmidt, 1990) and the momentum transfer (e.g. Devia et al., 1995) the basic data on flow fields and plume structure are still incomplete and no previous attempts at two-phase CFD-simulations in laboratory or industrial scale PS-processes were found in the literature.

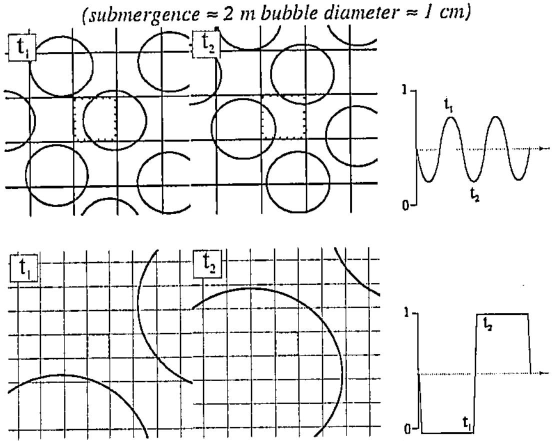
Due to similarity of the basic concept in ladle injection and PS-converting, the same methods used in the mathematical models of ladles were assumed to be suitable for a PS-converter as well. The ladle injection models can be divided into groups by the two-phase modelling technique. Alternative methods are Lagrangian two-phase approach, in which transport equations are solved for the liquid-phase and a bubble trajectory equation is solved for the gas-phase; and an Eulerian two-phase calculation procedure, in which transport equations are solved the both phases. These methods were tested with the built-in two-phase modelling options of the PHOENICS 2.1: the Lagrangian GENTRA-algorithm and the Eulerian IPSA-algorithm. In this work the IPSA-algorithm was chosen for the basic two-phase flow approach. The GENTRA algorithm was found to be unreliable in the studied gas injection cases, because it does not take into account the volume of fluid displaced by the gas bubbles. The volume fraction of gas was found to be locally very high in the studied cases.

Despite the apparent similarity of gas injection in a ladle and in a PS-converter, detailed analysis of bubble plume structures revealed some substantial discrepancies for the applicability of the Eulerian two-phase model to these processes. Due to the deep submergence of the nozzle, the bubble plume in a steel ladle is mostly a fully developed dispersion of relatively small bubbles, whereas most of the bubbles in a Peirce-Smith converter are very large, under formation or deforming due to their recent disengaging from the relatively low submerged tuyeres (e. g. Figure 2).

Due to the dimensions of a PS-converter, computational bubble size is inevitably larger than some of the computational cells, and not all the assumptions of inter-phase momentum transfer equations will be satisfied. Increasing the bubble size to larger than the cell dimensions leads to periodically time dependent situation as shown in Figure 1. If, in a steady state calculation, the volume fraction of gas is interpreted as probability of the extent of the gas phase at a given cell location and time, the solved gas phase velocity must be interpreted as time averaged velocity of periodically appearing bubbles. For example, in inter-phase momentum transport equations of the Eulerian steady state model, the drag force is calculated based on this average velocity of the gas phase. However, the drag force is a non-linear function of instant velocity, so the periodical nature of the flow should be taken into account in solving the total drag force. Also, it is presumed that the velocity of the dispersed phase in the inter-phase momentum transfer equations is always the same as the velocity of a discrete bubble. This is not always true. In the case of gas injection, the velocity of gas flowing into a growing bubble at a cell next to the nozzle is higher than the average velocity of the moving interface of the bubble in question, if the disengaged bubble is larger than the cell. Furthermore, most of the drag coefficients are experimentally defined for bubbles in a uniform flow field; this is not true in the mathematical model if the bubble surface exceeds the cell boundaries, unless velocity is constant in the cells surrounding the bubble.



An example of fully developed bubble plume in a ladle
(submergence ≈ 2 m bubble diameter ≈ 1 cm)



An example of primary bubbles in a PS-converter
(submergence ≈ 0.5 m bubble diameter ≈ 40 cm)

Fig 1. Schematic presentation of the relation of bubble and mesh dimensions to time dependency.

Another notable difference between PS-converters and ladles is the height of the spout formed on the free surface of the liquid bath. In a ladle it is only a few percent of the bubble plumes height. In a Peirce-Smith converter the height of the spout may be the same order as the submergence. Discharge of an undeveloped bubble jet of the PS-converter is probably a periodical phenomenon, while discharge of fully developed bubble jet in a ladle resembles more a steady state situation.

Solving the correct time dependent boundary conditions for each bubble in the PS-converter would demand a reliable free surface tracking method, and solving this kind of model for an industrial scale converter would take an enormous amount of CPU-time. Thus, any practical applications of a mathematical model of a PS-converter probably have to be time averaged. For this reason, a mathematical model was constructed with the same steady state Eulerian modelling methods that were used for the ladle injection cases, despite their theoretical drawbacks. However, due to recognised discrepancies of the processes, published experimental data on ladles were not seen as suitable for verifying the applied mathematical methods in the case of PS-converters. Thus, some physical modelling experiments were needed.

Furthermore, a parametric study of the model was necessary, because mathematical formulation of some typical properties of bubble jets is not yet well established in spite of the large number of published ladle injection models. The influence of some arguable formulas and variables was tested in the case of a laboratory scale ladle injection model as well as in the case of a laboratory scale PS-converter model.

2.2 Physical model

A water model was constructed for verifying the applicability of the two-phase modelling method used for the PS-process. The prepared water model was a ¼ scale slice of a typical industrial scale Peirce-Smith converter. The slice, presented in Figure 3, includes two nozzles. The blowing parameters of the model were scaled using dimensionless groups and specific magnitudes related to gas injection conditions. Dimensions and blowing parameters are summarised in Table 1.

The velocity field of the liquid was determined by measuring lengths of marker particle trajectories. Dimensions of the bubble jet were defined using still photographs, and average volume of disengaging bubbles was determined by measuring formation frequency acoustically and verifying this using video observations.

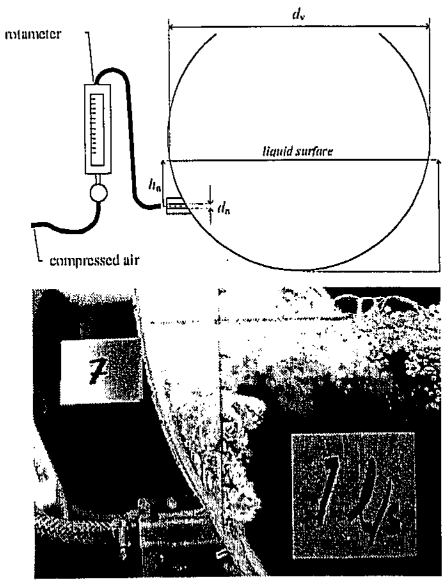


Fig 2. Schematic presentation of experimental apparatus and photographed example of bubble plume.

Table 1. Dimensions and blowing parameters of a typical PS-converter and the water model used for validation of the mathematical model.

	Converter	model
Injected gas	(28% O_2)	air
Volumetric flow rate (Nm^3/s)	7.55	0.004
density (kg/m^3)	1.2	1.2
Liquid	matte	water
mass (kg)	144 247	30.0
height h_b(m)	1.36	0.36
volume (m^3)	31.4	0.03
static pressure at tuyere level (Pa)	115 289	102 572
viscosity (50% Cu, Pas)	0.01	0.001
density (50% Cu, kg/m^3)	4600	998
temperature (K)	1493	293
Tuyeres		
number	42	2
spacing (m)	0.205	0.065
diameter d_n (m)	0.041	0.009
gas velocity (m/s)	138.5	31.5
submergence h_n (m)	0.52	0.13
Geometry		
inner diameter d_v (m)	3.46	0.86
length (m)	9.14	0.13
filling ratio	0.39	0.39
Dimensionless groups		
modified Froud number	12.4	12.5
Re number of the nozzle	475 155	23 847
Specific values		
specific volume flow rate (s^{-1})	0.23	0.13
specific power of buoyancy (W/kg)	2.07	0.34
specific power of gas kinetic energy (W/kg)	0.56	0.08

2.3 Mathematical model

2.3.1 *Solved Transport Equations*

The model does not yet include heat transfer. The solved governing equations were:

- continuity equation for gas and liquid phase
- time averaged liquid momentum conservation equation for all three components in cartesian co-ordinates
- time averaged gas momentum conservation equation for all three components in cartesian co-ordinates.

The standard k-ε model was used to calculate turbulence effects. It requires solving of

- transport equations of turbulent kinetic energy and dissipation of turbulent energy.

2.3.2 *Boundary Conditions for the Governing Equations*

An example of solution domain and computational mesh is presented in Figure 4. The computational domain is based on a slice of a cylinder. It covers, in the longitudinal direction, a section between two symmetry planes which were located in the middle of a tuyere, and in the middle of a space between two tuyeres. Because the free surface of the liquid is not aligned on a diameter of the PS-converter and the computational domain covers the spout region, a body fitted co-ordinate system was used instead of cylindrical co-ordinates. The distribution and number of nodes in the grid were optimised separately for each case depending on the dimensions of the simulated vessel and blowing parameters.

On the bottom edge of the domain, no-slip conditions were applied to momentum transfer equations, and logarithmic wall functions were imposed for the turbulence boundary conditions.

The gas was allowed to leave at the upper edge of the domain. This edge was assumed to be free surface of the liquid and to have no friction or interactions with turbulence.

The shape of the spout was obtained by eliminating the pressure gradients on the free surface. This was done by deforming manually the shape of the computational domain until pressure distribution was as uniform as possible on the upper edge.

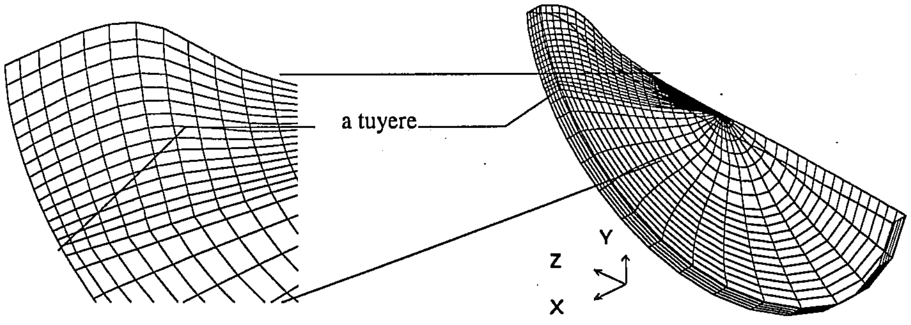


Fig 3. The solution domain for the PS-converter water model (number of cells = 5x30x15).

This "manual iteration" of the spout shape was carried out during the solution procedure of each calculated case.

For the boundary conditions, a circular tuyere was simplified to a square with the same cross-sectional area. Horizontal gas velocity boundary conditions for the tuyere were calculated directly from the volume flow rate and the pressure boundary conditions from the mass flow rate of the gas injection. Vertical gas velocity at the tuyere outlet was set to zero. Tuyere outlet was assumed to be filled with gas phase, so liquid velocities, turbulent kinetic energy and dissipation were also set to zero.

2.3.3 *Momentum Transport between Phases*

The drag force was taken into account by source terms in the momentum transport equations. The source terms were calculated from the volume fraction of gas and the integral drag force of a discrete spherical bubble. The drag coefficient was allowed to vary in the calculations depending on the shape region determined by dimensionless groups, but the bubble shape was always in the range defined for spherical-cap-shaped bubbles, for which the drag coefficient is a constant value of 8/3 (Clift et al., 1978).

In the Eulerian two-phase models, particle size and shape distribution have to be simplified to mean diameter of a spherical bubble. The bubble diameter was set for the ladle cases according to the mean diameter of a volume equivalent sphere measured by Castillejos and Brimacombe, 1987, and Sheng and Irons, 1992. The bubble size range in ladle models is typical of a developed dispersion. In the PS-cases the mean bubble size was assumed to be in a range typical of the primary bubble zone, because the disintegration process of the bubbles does not have time to be completed before the bubbles reach the free surface of the bath. Several primary bubble volume equations suggested by Davidson and Schüler, 1960, Irons and Guthrie, 1980[1], Hoefele and Brimacombe, 1979, and Brimacombe et al., 1991[2] were tested. The best agreement of calculated bubble volume and measured bubble formation frequency was obtained with Davidson and Schüler's original equation

$$V = 1.378 Q^{\frac{6}{5}} g^{-\frac{3}{5}} \qquad (1)$$

in the case of a laboratory scale water model as well as in the case of a real PS-converter and, it was applied to define the diameter of a volume equivalent bubble in the PS-converter cases.

Because of the significance of the bubble size, as a variable in inter-phase momentum transfer equations and simplifications made in the mathematical model, the sensitivity of the model to bubble size was studied. For the PS-converter case, the bubble size was found to be the most significant arguable variable of the mathematical formulation. In the case of ladle injection, influence of the bubble size was more difficult to evaluate, because of more complex interactions of turbulence and inter-phase momentum transfer.

2.3.4 *Interaction of Bubbles and Turbulence*

Relatively large bubbles are known to increase the intensity of turbulence in the liquid phase. Because there is no uniform and generally accepted mathematical formulation for bubble induced turbulence, different kinds of turbulence models presented in the literature were tested. All the models were modifications of the standard k-ε model. Three of these models presented in Schwarz, 1996, CHAM, 1995, and Sheng and Irons, 1993 were based on source terms attached to transport equations of turbulence, and one presented in CHAM, 1995 was based on direct modification of turbulent viscosity.

All the results calculated with additional source terms were more reasonable than the ones calculated with the bubble-induced viscosity. Tested source terms were found to have a considerable and similar influence on calculated properties of bubble plumes in ladle models. By using an appropriate combination of additional source terms and coefficients, a good correlation with calculations and measurements was obtained. Unfortunately, calculated results were found to be sensitive to coefficients source term equations include, and relation of the coefficients to flow conditions

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[1] equation (4)

[2] equation (2.14)

is not generally known. However, in the case of a PS-converter, poorly established coefficients of the modified turbulence models are not expected to influence the reliability of the model, because modelling of bubble-induced turbulence did not have a notable impact on calculated bubble or liquid velocities.

On the other hand, the liquid phase turbulence influences the motion of the dispersed phase. Fluctuating velocity components transfer bubbles in directions divergent from their mean direction of motion, and consequently the bubble plume spreads out. The ratio of this turbulent diffusivity of dispersed phase to eddy diffusivity is a function of particle response time and microscales of turbulence (Soo, 1990). It could be expected that significant turbulent diffusion of bubbles requires that the typical scale of the turbulence is larger than bubble size or at least in the same range. In the Eulerian two-phase models, this is considered in the additional diffusive terms of the continuity equations. The magnitude of the turbulent dispersion is determined by turbulent diffusion coefficient which can be presented by the turbulent Prandtl number of the void fraction. Most of the diffusion coefficient values proposed in the literature are based on trial and error experiments and they cannot be expected to be valid in a general case. The trial for finding an appropriate diffusion coefficient was started with a Prandtl number of 0.1 as suggested by Türkoglu and Farouk, 1990. Changing this value had a noticeable influence on the results calculated with the ladle models, but very limited impact in the case of a PS-converter. The reason for the difference between these cases is assumed to be the higher volume fraction of gas in the PS-converter case. It is also presumed that there is a connection between the insensitivity of the PS-converter model to bubble induced turbulence and to the turbulent diffusion coefficient. Supposedly, bubble induced turbulence exerts an influence on properties of the bubble plume mostly by increasing turbulent dispersion. If the diffusion term in a void fraction transport equation is negligible, bubble-induced turbulence models or turbulent diffusion coefficient do not have a noticeable influence on calculated flow velocities or void fractions of the bubble plume in question as is the situation in the case of a PS-converter.

3. VERIFICATION OF ANALYSIS METHODS

The laboratory scale water model described above was used for verifying the applicability of modelling principles for the PS-converter. Unfortunately, proper equipment was not available to measure bubble and liquid velocities or volume fractions inside the bubble jet, so the model was verified by comparing the measured and calculated velocity fields of liquid outside bubble plume, and the general dimensions of the plume. Figure 4 shows example of a measured and a calculated velocity field. Measurements are assumed to contain a slight systematic error caused by a small density difference between water and the tracer particles. This error should not affect the measured horizontal velocities at the vertical diameter which are compared with the calculated ones in Figure 5. In all tested blow rates, the disagreement between predictions and measurements was estimated to be within the limits of measuring accuracy.

The reliability of the predicted gas distribution was evaluated by comparing the dimensions of the bubble plume in photographs with 1/8 s exposure time and calculated void fraction contours (Figures 6 and 7). Qualitative comparison can not be made by this method, but the calculated shape and size of the bubble plume was shown to be quantitatively in good agreement with the real one.

4. PREDICTED FLUID DYNAMICS OF AN INDUSTRIAL SCALE PS-CONVERTER

Although the mathematical model is under development and still lacks a very important factor, namely heat transfer, some calculations were made to analyse the flow conditions in an industrial scale Peirce-Smith converter and to evaluate scaling factors of the physical models of PS-converters.

The dimensions of the grid constructed for the water model were scaled up to an industrial scale PS-converter (Table 1). Comparison of calculated results, obtained with properties of water and properties of matte, revealed that the modified Froude number did not fulfil the geometric similarity criteria of the bubble plumes when it was used to scale blowing speed.

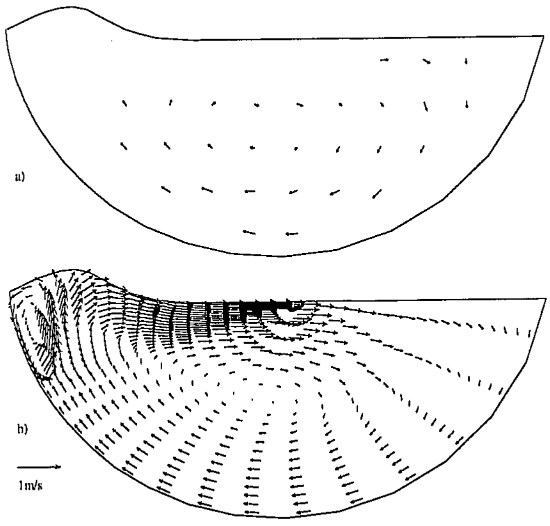

Fig 4. Measured liquid velocities a) compared with predicted velocity field b) of PS-converter water model (blow rate 0.004 m^3/s).

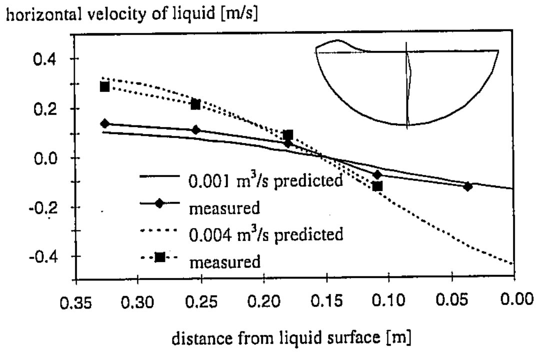

Fig 5. Comparison of predicted and measured horizontal velocity of liquid at the vertical diameter of water model with different blow rates.

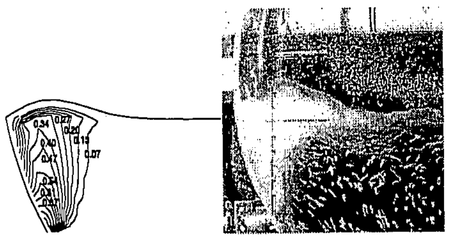

Fig 6. Calculated void fraction contours and photographed bubble plume for blow rate 0.001 m^3/s.

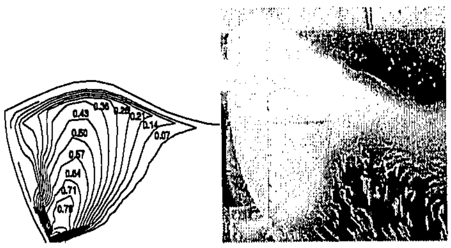

Fig 7. Calculated void fraction contours and photographed bubble plume for blow rate 0.004 m^3/s.

Penetration of the bubble plume decreases as the density of the fluid increases. In the cases with very dense liquid (8000 kg/m^3), the vortex behind the bubble plume tends to disappear and the bubble plume becomes attached to the converter wall. Besides inertia and gravitation, the vortex (Figure 4b) seems to have an important role in the penetration of the bubble jet. According to the results calculated in this study, it does not seem probable that jet penetration is a function just of injection and nozzle parameters. The geometry of the vessel is also expected to have a significant influence on penetration.

It was also noticed that geometric or kinematic similarity in the water model and in the real PS-converter would require a substantially higher blow rate than used in this study (compare Figures 7 and 8). It was estimated that similarity criteria could be satisfied in the water model with a blow rate which corresponds to a lower specific power[1] and a higher specific volume flow rate than used in the real process. However, this does not concern the impact of thermal energy.

Figure 8 shows that the calculated bubble plume in the PS-converter is relatively narrow. Short penetration and low submergence lead to very high volume fractions of gas in the plume core. Because the volume fraction of matte is substantially low in most of the area of the two-phase region, mass transfer of oxygen is unlikely to be the limiting factor of the reaction rate in the bubble plume as has been suggested by e. g. Brimacombe et al., 1985. The calculated volume fraction of gas supports the assumption of Adjei and Richards, 1991 that the extent of reactions during bubble formation is a very important factor in overall reaction rate.

The main reason for a poor penetration of the bubble jet is probably the strong buoyancy force due to the large density difference of the fluids and the relatively high momentum of the recircular flow. Figure 9 shows that the velocity of a circular flow loop is about two metres per second.

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[1] Specific power of buoyancy presented by Brimacombe et al., 1985

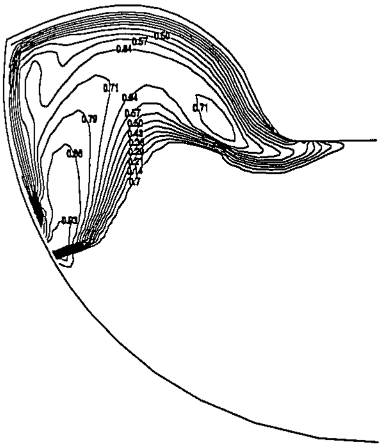

Fig 8. The predicted contours of void fractions on a tuyere plane in an industrial PS-converter.

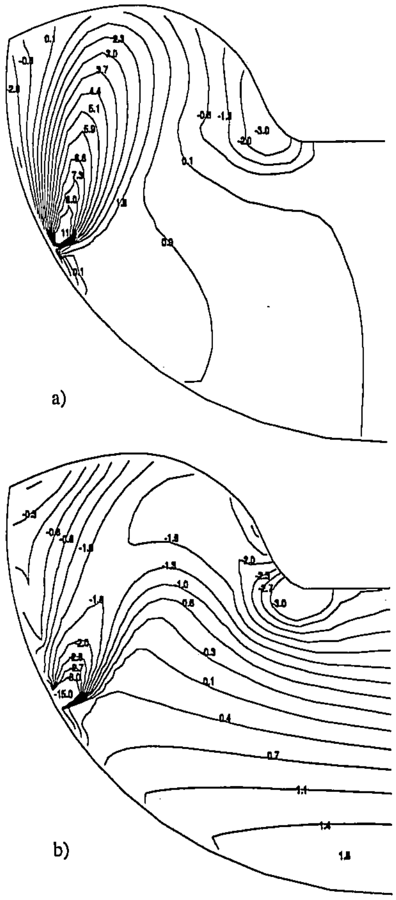

Fig 9. Predicted vertical a) and horizontal b) velocity of liquid on a tuyere plane in an industrial PS-converter.

5. CONCLUSIONS

In this study, fluid dynamics of the Peirce-Smith converter was simulated with a commercial CFD-code. The predicted flow conditions of the PS-converter water model obtained with the Eulerian two-phase model were compared with the measurements made using the physical model and with the calculations made with the ladle injection model in order to evaluate the applicability of the mathematical formulation to the PS-process. For studying flow phenomena in a real process, an analysis of the general pattern of isothermal flow and the bubble plume structure was made with a full-scale mathematical model. According to the results, the following conclusions were drawn.

- Experimental measurements corresponded surprisingly well with calculated results, considering the simplifications made in the mathematical model. The fact that the periodic nature of the flow was not taken into account in the mathematical model did not seem to have any noticeable impact on the calculated overall momentum transported between the phases.
- The mathematical model was found to be insensitive to bubble-induced turbulence and the turbulent diffusion coefficient of dispersion in the case of PS-converting, in contrast to the case of the ladle injection. The computational bubble size was found to be the only inadequately established variable, which had an influence on the velocities and the void fractions calculated for the PS-converter water model. This was probably a consequence of the high volume fraction of gas in the bubble plume of the PS-converter water model.
- Generally, the results obtained with the present mathematical model encourage the utilisation of the applied methods in further developments of the model. Estimated flow fields are assumed to be accurate enough to utilise them as the velocity boundary conditions for more detailed models, including e.g. dynamic free surfaces, heat and mass transfer, and chemical reactions.
- The predictions made with the cold model of an industrial scale Peirce-Smith converter revealed that similarity criteria in the physical models of the process would be satisfied better with different scaling factors than the ones used in this study. According to the calculated results, the bubble plume in a real process has sharper distri-

butions of void fractions and penetrates into the melt less than in the physical modelling experiments.

- Calculated volume fraction of gas in the core of the bubble plume was so high that it indicated more a froth or a foam, rather than a flow of bubble dispersion. The low volume fraction of matte in the core of the bubble plume possibly leads to a locally high oxidation rate of matte and re-reduction of oxidised matte in the recirculatory flows. The best conditions for oxidation reaction seem to be near the tuyere, where recently discharged gas meets the recirculatory flow of matte. Thus, probably a large part of the oxidation reactions occur during the bubble formation.

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REFERENCES

Adjei, E., Richards, G.G., 1991, "Physical Modelling of Mass Transfer in a Peirce-Smith Converter", *Copper 91 - Cobre 91,* vol. IV, Ottawa, Pergamon Press Inc., pp. 378-388.

Brimacombe, J. K., Bustos A. A., Jorgensen, D. and Richards, G. G., 1985, "Toward a Basic Understanding of Injection Phenomena in the Copper Converter", *Physical Chemistry of Extractive Metallurgy,* Warrendale, TMS-AIME, pp. 327-352.

Brimacombe, J. K., Meredith. S. E., Lee R. G. H., 1984, "High Pressure Injection of air into a Peirce-Smith Copper Converter", *Advances in Sulfide Smelting,* TMS-AIME, Warrendale, pp. 841-854.

Brimacombe, J. K., Nakanishi, K., Anagbo, P. E., Richards, G. G., 1991, "Process Dynamics: Gas-Liquid", *Proceedings of the Elliot Symposium on Chemical Process Metallurgy,* Iron and Steel Society Inc., Warrendale, pp. 343-412.

Bustos A. A., Richards, G. G., Gray, N. B. and Brimacombe, J. K., 1984, "Injection Phenomena in Nonferrous Processes", *Metall. Trans. B,* 15B, pp. 77-89.

Castillejos, A. H. and Brimacombe, J. K., 1987, "Measurement of Physical Characteristics of Bubbles in Gas-Liquid Plumes: Part II", *Metall. Trans. B,* 18B, pp. 659-671.

CHAM, 1995, *The PHOENICS on-line information system,* Concentration, Heat and Momentum Ltd., London.

Clift, R., Grace, J. R. *and* Weber, M. E., 1978, *Bubbles, Drops and Particles,* Academic Press Inc., New York, p. 380.

Davidson, J. F. and Schüler, B. O .G, 1960, "Bubble Formation at an Orifice in a Viscous Liquid", *Trans. Instn. Chem. Engrs., 38,* pp. 144-154.

Devia, M. I., Fuentes, R. H. and Guevara, F. J., 1995, "Gasenous injection phenomenology in Peirce-Smith converters", *Copper 95 - Cobre 95,* vol. IV, The Metallurgical Society of CIM, pp. 239-253.

Garrido, G. F. and Lee, R. G. H, 1987, " Optimizing Oxygen Enrichment of High-Pressure Air Injection into Copper Converters", *Copper 87 - Cobre 87,* vol. IV, Pergamon Press Inc., Ottawa, pp. 1-17.

Hein, K and Schmidt J., 1990, "Einfluss der Einblas- parameter auf Strömungs-verhältnisse und Stof- fübergang in einem Peirce-Smith-Konverter", *Metall,* 44, pp. 1062-1066.

Hoefele, E.O. and Brimacombe, J.K, 1979, "Flow Regimes in Submerged Gas Injection", *Metall. Trans. B,* 10B, pp. 631-648.

Iguchi, M. Ueda, H. and Uemura, T., 1995, "Bubble and Liquid Flow Characteristics in a Vertical Bubbling Jet", *Int. J. Mult. Flow,* 21, pp. 861-873.

Ilegbusi, O. J. and Szekely, J., 1990, "The Modeling of Gas-bubble Driven Circulation Systems", *ISIJ Int.,* 30, pp. 731-739 I II

Irons, G. A. and Guthrie R. I. L., 1980, "Bubbling Behaviour in Molten Metals", *Can. Met. Quart.,* 19, pp.381-387.

Johansen, S. T., Robertson, D. G. C., Woje, K. and Engh, T. A., 1988, "Fluid Dynamics in Bubble Stirred Ladles: Part I. Experiments", *Metall. Trans. B,* 19B, pp. 745-754.

Jönsson, P. and Jonsson, L., 1996, "Modelling of Fluid Flow Conditions around the Slag/Metal Interface in a Gas-stirred Ladle", *ISIJ Int.,* 36, pp. 1127-1134.

Kimura, T., Tsuyuguchi, S., Ojima, Y., Mori, Y. A Ishii, Y., 1986, "Refractory Protection by high Speed Blowing in a PS Converter", *JOM, 38,* pp. 38-42.

Liow, J-L., Gray, N. B., 1990, "Slopping Resulting from Gas Injection in a Peirce-Smith Converter: The Period of the Standing Wave", *Metall. Trans. B,* 21B, pp. 657-664.

Mazumdar, D. and Guthrie, R. I. L., 1995, "The Physical and Mathematical Modelling of Gas Stirred Ladle System *"ISIJ Int.,* 35, pp. 1-20.

Schwarz, M. P., 1996, "Simulation of gas injection into liquid melts", *Appl. Math. Modelling,* 20, pp. 41-51.

Sheng, Y. Y. and Irons G. A., 1992, "Measurements of the internal Structure of Gas -Liquid Plumes", *Metall. Trans. B,* 23B, pp. 779-788.

Sheng, Y. Y. and Irons G. A., 1993, "Measurement and Modeling of Turbulence in the Gas/Liquid Two-Phase Zone During Gas Injection", *Metall. Trans. B,* 24B, pp. 695-705.

Sheng, Y. Y. and Irons G. A., 1995, "The Impact of Bubble Dynamics on the Flow in Plumes of Ladle Water Models", *Metall. Trans. B,* 26B, pp. 625-635.

Soo, S. L., 1990, *Multiphase Fluid Dynamics,* Science Press, Beijing, p. 691.

Türkoglu, H. and Farouk, B., 1990, "Numerical Computations of Fluid Flow and Heat Transfer in a Gas-Stirred Liquid Bath", *Metall. Trans. B,* 21B, pp.771-781

Türkoglu, H. and Farouk, B., 1991, "Mixing Time and Liquid Circulation Rate in Steelmaking Ladles with Vertical Gas Injection", *ISIJ Int.,* 31, pp. 1371-1380.