A CFD MODEL FOR DENSE MEDIUM CYCLONES

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ABSTRACT
In this paper, a CFD modelling strategy is described in detail. Then the paper focuses on studying the effect of the non-Newtonian rheology and turbulence phenomenon in the medium modelling. Therefore, fluid flow modelling is carried out for the Herschel-Bulkley and Power Law rheological models and compared with the Newtonian model with and without turbulence. Results for axial and velocity profiles show that the turbulence effect is more important than the non-Newtonian rheology. Consequently, a Newtonian model is developed for the medium for the case where the segregation of the magnetite particles in the medium can be neglected. Also a model for the coal particles is presented and combined with the medium model to give a practical model for DMCs. The model is an Eulerian-Lagrangian approach. Finally the model is used to evaluate the influence of coal particles size on the separation process. Coal particles of 5 mm and 1 mm diameter were considered. Predictions show the degradation of the efficiency of separation for smaller coal sizes. Also low density coal particles are initially affected by the recirculation region. Finally, higher residence times were found for neutrally-buoyant particles

NOMENCLATURE
\( u_i \) velocity component in \( i \) direction
\( p \) pressure
\( g_i \) gravity component in \( i \) direction
\( v \) kinematic viscosity
\( \mu \) dynamic viscosity
\( \mu_l \) dynamic laminar viscosity
\( \rho \) density of the fluid
\( S \) strain rate
\( \tau \) shear strain
\( \tau_y \) yield strain
\( \rho_p \) density of the particle
\( C_D \) drag coefficient
\( D_p \) diameter of a particle
\( r \) coefficient of correlation
\( u_{pi} \) velocity of the particle in \( i \) direction

INTRODUCTION
Dense medium cyclones (DMCs) are widely used coal-cleaning devices in the mining industry. For hard-to-clean (+10% near gravity material) in the size range of 50 mm to 0.5 mm, DMCs are very effective. In operation, the raw coal and the medium are fed at a precise pressure into the tangential inlet. The ensuing flow spirals towards the apex of the device. In the core of the cyclone, a very fast upward flow is created. Centrifugal classification causes shale to move outwards towards the inner wall of the conical shell. As a result, shale is discharged from the apex and coal is carried by the rising internal spiralling flow towards the vortex finder to be discharged from the overflow.

Experimental studies, empirical and analytical modelling and computational modelling of the fluid flow pattern, pressure drop, and solids motion in hydrocyclones have been carried out by many researchers (Kesall, 1952; Pericleous and Rhodes, 1986; Hsieh, 1988; Davidson, 1994; Devallapalli and Rajaman, 1996; He, 1999). However, for DMCs such information is very limited. DMCs have been mainly subjected to experimental and analytical studies (Davis, 1987; Napier-Munn, 1990; Wood, 1990). Within the experimental studies, only parametric studies analysing the influence of geometrical and operating variables on the efficiency of separation are reported. The utility of this information is limited. Reported computational modelling of DMCs is virtually non-existent (only Zughbi et al, 1991). Having a reliable computational model for DMCs can assist in the improvement of the design and operating conditions, essentially enhancing the experimental approach.

Within the research context of improving the understanding of DMCs by means of computational simulation, this paper discusses our modelling approach and analyses the effect of the non-Newtonian character of the medium. Having found that a Newtonian approximation for the medium is valid, a CFD model for DMCs is proposed based on a composite medium with ultrafine magnetite. The CFD commercial code FLUENT is used to construct the CFD model.

MODEL FORMULATION
Fluid flow and particle interactions in DMCs are very complicated phenomena. Broadly speaking three phases can be distinguish. They are water, solids (magnetite, coal and waste particles) and air. However, from a numerical point of view different density and size particles must be considered as different phases. Hence, the complexity of constructing a model for DMCs is obvious; and a simplified treatment of the phenomena is appropriate.

Under normal operation, the vortex flow within the cyclone body generates an air core along the entire axis of the body. The air core has been modelled to be approximately cylindrical in shape and axially centred (Suasnabar and Fletcher, 1998). As a consequence, the air-fluid interface can be assumed fixed and may be numerically approximated as a slip wall. Also for a particular DMC geometry, modelling of the slurry at different specific gravities can be carried out with the
same fixed air core diameter, since Wood (1990) found that the air core shape and size are independent of the density of the slurry.

With the simplified treatment of the air core, further simplifications are required for the fluid composed of water, magnetite particles and raw coal particles. For modelling purposes, the magnetite-water mixture may be separated from the raw coal particles. Then separate models can be formulated for the medium and coal particles. However, the models must interact with each other to produce an overall model for DMCs.

Modelling of the raw coal particles may be a difficult task. Since the range of sizes and densities of the particles is quite large it is difficult to use an Eulerian continuum model. Additionally, in order to reproduce partition curves, a real representation of the raw coal particles cannot be obtained by an Eulerian model due to computational limitations. Eulerian models may only be used to evaluate one or two monosize and/or monodensity coal particle separations at very high concentrations. Thus the Lagrangian approach appears to be the more appropriate alternative. Since mainly particle-particle interactions are difficult to account for in a Lagrangian reference frame, only dilute concentration calculations may be accomplished. However, if clouds of particles instead of single particles are tracked (Devulapalli and Rajamani, 1996), a high concentration of coal can be considered. In the latter model, concentration profiles within the cloud are approximately represented by a probability density function (pdf). Coal particle tracking at dilute concentration can be considered an approximation of the density tracer method (Davis, 1985). In this paper, only a model for coal particles at dilute concentrations is presented. Future models will consider more comprehensive models.

**Non-Newtonian Models**

For Newtonian fluids, the shear stress is proportional to the strain rate ($\tau = \mu \dot{S}$). Where $\mu$ is independent of $\dot{S}$. $\dot{S}$ is defined in general form as:

$$\dot{S} = \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

(1)

For non-Newtonian fluids, the viscosity becomes a function of $\dot{S}$, and is described by the variable $\eta$, where $\tau = [\eta(\dot{S})] \dot{S}$. Mathematical relations for $\eta(\dot{S})$ known as rheological models are abundant. In this paper modelling the fluid flow in dense medium cyclones in a laminar fashion and considering the Herschel-Bulkley and the Power Law models was carried out. The former is described by Equation (2). The Power Law results when the yield strain to is neglected.

$$\tau = \tau_y + (k^{\dot{S}^{n+1}}) \dot{S}$$

(2)

In a three-dimensional model for cyclones, there are six components of the shear strain. Considering shear-dominated flow, the normal strains are neglected. Additionally, experimental measurements in hydrocyclones have shown that the velocity gradient in the tangential direction for regions below the vortex finder is very small. So only the radial-axial strain ($\tau_{xy}$) and axial-tangential strain ($\tau_{y0}$) are significant. Then an axisymmetric...
approximation is justified for analysing the non-Newtonian effect of the fluid flow modelling.

**Experimental Studies**

It is widely reported that the magnetite medium is a non-Newtonian fluid. According to He (1994), the best rheological model was found to be the Casson model. Also, he found that the Herschel-Bulkley model also correlates well his experimental data, though he preferred the Casson model due to the physical significance of the Casson viscosity. Laskowsky (1994) conducted shear rate-shear strain measurements for different types of magnetite medium. Results for a medium of ultratine magnetite is reported in Figure 2 and 3.

![Figure 2](image)

**Figure 2:** Curve fitting with different rheological models for shear strain vs strain rate experimental data of an ultrafine magnetite medium with 1.5 s.g.

According to Figure 2, in fact the Casson model represent best the ultrafine magnetite medium (r=0.995). Nevertheless, also the Herschell-Bulkley model correlates well the experimental data (r=0.985). With this model, for strain rate above 200 1/s, higher discrepancies are observed when the strain rate increases. A model with zero yield stress such as the Power Law model can also approximately fit the non-Newtonian data (r=0.980).

![Figure 3](image)

**Figure 3:** Shear strain vs strain rate measurements at different specific gravities.

If a Newtonian model is used to model the medium flow, then an equivalent Newtonian viscosity is required. This viscosity must be chosen to represent the whole range of the strain rate. Therefore, if a Bingham Plastic model is formulated and its yield stress neglected, the remaining equation gives the equivalent Newtonian viscosity. This viscosity is approximately constant at different densities according to Figure 3. Consequently, in the event of small segregation of magnetite particles in the medium, this viscosity can still be used.

**Non-Newtonian Rheology and Turbulence Effect**

Assuming axisymmetric geometry and considering the medium as a homogenous fluid, fluid flow calculations were carried out with two different rheological models. Additional calculations were performed for a Newtonian fluid for laminar and turbulent flows. The turbulent model is based on a Reynolds stress model (RSM) described below. Results in terms of axial and tangential velocity profiles at 106 mm and 270 mm form the cyclone roof are presented in Figures 4 and 5.

![Figure 4](image)

**Figure 4:** Effect of non-Newtonian models and turbulence on axial velocity profiles at 106 mm (top) and 270 mm from the cyclone roof.

![Figure 5](image)

**Figure 5:** Effect of non-Newtonian models and turbulence on tangential velocity Profiles at 106 mm (top) 270 mm from the cyclone roof.

It can be noted that the axial and velocity profiles at the two locations for the non-Newtonian models and
Newtonian laminar are very similar. On the contrary, velocity profiles considering turbulence differ from the others. Therefore, turbulence appears to be more significant than the non-Newtonian rheology.

Consequently, if we consider the total viscosity for the fluid to be composed of the laminar, turbulent and non-Newtonian contributions, clearly the turbulence viscosity will be dominant, over-riding the influence of the others. So, a Newtonian turbulent fluid flow approximation for the dense medium behaviour is justified, provided that the turbulence is significant.

Some of the implications of this assumption in the coal separation process may be uncertain. For example, for the medium that follows the Herschel-Bulkley model, coal particles will need to overcome the yield stress to begin moving. This behaviour is expected in a laminar flow, however, in turbulent flows, the influence of the yield stress is unknown. Turbulence may also play a key role in suppressing the influence of the yield stress in the separation of coal particles.

A MODEL FOR DENSE MEDIUM CYCLONES

A model for dense medium cyclones is proposed. The model is an Eulerian-Lagrangian combination. The medium is considered a Newtonian homogenous turbulent fluid and is modelled in an Eulerian reference frame. A Reynolds stress model is used for turbulence modelling. Coal particles are modelled in a Lagrangian reference frame. Turbulence dispersion of the coal particles is considered with a stochastic model. The model is valid when the segregation effect of the magnetite particles is neglected and coal particles are present at dilute concentrations. This implies that the model is for dense medium cyclones operating at high densities and for high volumetric medium/coal ratio. This particular case is for a medium made of ultrafine magnetite particles dispersed in water with 1.6 s.g.

The implications of the segregation phenomena in the simplified model may be neglected considering that the model was built for a medium with ultrafine magnetite. Additionally, in practice higher viscosities of the medium are expected due to contamination of the medium by clay and other fine contaminants (Napier-Munn, 1990). Additionally, partition curves constructed by tracking coal particles in a Lagrangian reference frame may be equivalent to the one obtained experimentally by density tracer techniques when the dense medium cyclone is operating with only medium.

A Model for the Magnetite Medium Flow

The model is for a standard 350 mm DMC and can be easily extended to other sizes and geometries. In the medium modelling, the magnetite medium is assumed to be a homogenous Newtonian fluid. The air core diameter was estimated following the approach of Smith (1962) and its boundary treated as slip wall. A cylindrical mesh of 27x28x42 was used. Since the volumetric flow rate at the tangential inlet and the cross-section area are known, the tangential velocity is easily calculated. The apex flow rate is estimated with the algebraic expressions of Wood (1990). Then outlet conditions are applied to both the vortex flow and apex. For this type of boundary condition, the axial velocity is prescribed to be uniform with a value determined from the flow rate. Turbulent quantities are assumed and input as turbulence intensity (5%) and characteristic length (diameter of the inlet/exits). From these quantities, the kinetic energy and eddy dissipation rates at the inlet/exits are calculated. Near wall calculation was carried out with a standard wall function (Rodi, 1980). The near-wall values of the Reynolds stresses and $\epsilon$ are computed from this function.

For the region far from the walls, the governing equations in their cylindrical form were solved. Due to limited space, only the continuity and momentum equations in their general form are presented here.

$$\frac{\partial u_i}{\partial x_i} = 0$$  \hspace{1cm} (3)

$$\frac{\partial}{\partial x_j}(\rho u_i u_j) = \frac{\partial}{\partial x_i} \left( \mu \frac{\partial u_i}{\partial x_j} - \rho u_i u_j \right) + \rho g_i$$ \hspace{1cm} (4)

where $\rho$ is the density of the fluid, $u_i$ is the mean velocity component in the $i$ direction, $\rho$ the fluctuating component of the velocity, $g_i$ is the gravitational acceleration and $p$ is the static pressure, respectively.

For steady-state, the transport equation for the Reynolds stresses can be expressed in symbolic form as:

$$C_i = P_i + D_i + \phi_i - \epsilon_i$$  \hspace{1cm} (5)

where:

$$C_i = u_i \frac{\partial}{\partial x_i} \overline{u_i u_j}$$ is the convective transport term,

$$P_i = \left( \frac{\partial}{\partial x_i} \frac{\partial u_i}{\partial x_j} \right) + \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j}$$ is the production term,

$$D_i = \frac{\partial}{\partial x_i} \left( \overline{u_i u_j} \overline{u_i u_j} \right) \frac{1}{\rho} \frac{\partial}{\partial x_i} (\rho \overline{u_i u_j} - \overline{u_i u_j} \overline{u_i u_j})$$ is the diffusion term,

$$\phi_i = \frac{1}{\rho} \frac{\partial}{\partial x_i} \overline{u_i u_i}$$ is the pressure-strain term,

$$\epsilon_i = 2 \nu \frac{\partial u_i}{\partial x_i} \frac{\partial u_i}{\partial x_i}$$ is the dissipation term.

The Reynolds Stress turbulence model requires solving the six transport equations for the individual stresses $\overline{u_i u_j}$ plus a transport equation for the dissipation rate, which is similar to the transport equation for the dissipation rate in the standard $k-\epsilon$ model. The diffusion, the pressure-strain and dissipation terms are modelled. The detailed analysis and approximations of the several terms within the transport equations are described in more detail by Launder et al.(1975). Summarising, assuming isotropy of turbulence, the diffusion term is described using a scalar diffusion coefficient, the wall-reflection term is neglected and the dissipation term is assumed isotropic.

Calculations were carried out for the DMC inclined 15° with respect to the horizontal. A medium of 1.6 s.g density and 3.3 cp viscosity was used. Predicted axial and radial velocity vectors for a planes axially centred and
perpendicular to the inlet are shown in Figure 6. From this figure it can be clearly seen the outer downward flow and the inner upward flow. The interaction of both flows generate the recirculation zones near the vortex finder region. The short-circuiting flow in the vortex region is also clearly shown.

Figure 6: Axial and radial velocity vectors at perpendicular plane to the inlet in a standard 350 mm DMC.

A Model for the Raw Coal Particles Separation

Coal particles are modelled in a Lagrangian reference frame. The trajectory of an individual particle is predicted by integrating the force balance on the particle. For the i-direction in general coordinates, the equation is:

\[
du_i = \frac{1}{T} \frac{du}{dt} + \left( \frac{2w_i u_i}{T} \right)^{1/2} \]

where \( w_i \) is a Gaussian distributed random number and \( T \) is the integral time. \( T \) is approximated as \( T_L = 0.3 \varepsilon k^{1/2} \), where \( T_L \) is the medium Lagrangian integral time. In order to improve the accuracy, the maximum time step is limited to be less than 0.3 \( T \). The trajectory of each particle is computed 10 times. Finally under the assumption of dilute concentrations for coal particles only one-way coupling is considered.

Results in terms of partition curves for two coal particle sizes are shown in Figure 7. Degradation of the separation efficiency for finer coal particles can be seen. Also, the density of separation for finer particles is higher. Finally the displacement of the particles (coal and waste) in the undesired stream are more evident when fine particles are treated.

![Figure 7: Partition curves for 5mm and 1mm coal particles in a 350 mm DMC.](image)

To predict the dispersion of the particles due to turbulence, the fluctuating medium velocity components are obtained by solution of the Langevin equation (Thomson, 1987), generalised for 3-dimensional flow as:

\[
u = \frac{1}{T} \frac{d\nu}{dt} + \left( \frac{2w \nu}{T} \right)^{1/2}
\]

where the constants are given by Morsi and Alexander (1972). Equation (6) is linearised and a variable time step Runga-Kutta technique for integration of the particle equation of motion is used. This procedure selects the integration time step based on the residence time of the particle within each medium control volume. The maximum time steps was fixed to be 20,000.

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<table>
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<th>Case</th>
<th>( \rho )</th>
<th>( t )</th>
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<tbody>
<tr>
<td>1</td>
<td>1.55</td>
<td>3.7</td>
</tr>
<tr>
<td>2</td>
<td>1.60</td>
<td>5.8</td>
</tr>
<tr>
<td>3</td>
<td>1.65</td>
<td>2.3</td>
</tr>
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Table 1: Residence time for 5 mm coal particles of different densities.
Figure 8: 5 mm diameter coal particles trajectory of different densities: 1.55 s.g. (left), 1.60 s.g. (middle) and 1.65 s.g. (right).

CONCLUSION

The effect of the non-Newtonian rheology of the medium and turbulence phenomenon in the modelling process was analysed. It was found that the turbulence effect dominate the influence of the non-Newtonian rheology. Then, a model was constructed for the medium for a particular case where the segregation for magnetite can be neglected. Also a model for the coal particles at dilute concentrations was presented. As a result a model for DMCs was proposed. The model is a combination of an Eulerian model for the medium and a Lagrangian approach for the coal particles. Finally given the framework for future developments, a more comprehensive model is being devised.

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REFERENCES


