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PSEUDO-SHOCK IN SUPERSONIC INJECTION FEEDER

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ABSTRACT

The flow in a Supersonic Injection Feeder [1,2] involves relatively thick boundary layers in a narrow channel. When the pressures at the extremities of such a duct are adjusted to produce a compression shock, the shock structure is radically different from a plane discontinuity. This difference arises solely due to shock wave-boundary layer interaction [3], and gives rise to the so-called "pseudo-shock". In this paper, results of CFD simulations of a pseudo-shock in clean gas (air) are compared with predictions of the "Diffusion" model [11], the "Modified-Fanno" model [1,15] and with experimental results [1, 7]. An analysis of the effect of small particles on pseudo-shock structure is offered in the form of extensions of the analytical models and CFD simulations.

INTRODUCTION

Supersonic Injection Feeder

A Supersonic Injection Feeder [1,2] is a pneumatic device designed to convey dry particulate matter to high-pressure destinations. A zone of relatively low pressure is created in a supersonic gas stream, and particulate matter in the form of a gas-particle suspension is introduced into it. The particles in the combined stream are then conveyed to the high-pressure destination via a compression shock. A schematic diagram is shown in Figure 1. Ideally, the shock is a plane discontinuity with an abrupt change in flow parameters across it.

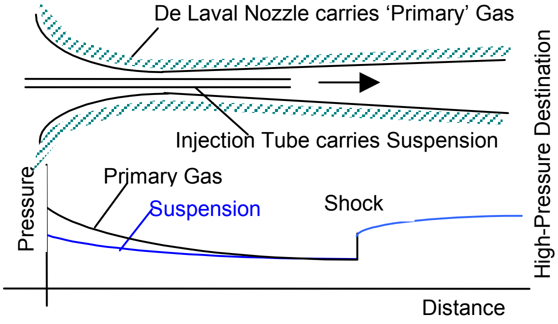

Figure 1 Flow in Supersonic Injector

The Pseudo-Shock

In reality, however, there is a severe interaction between the shock and the shear layers growing along the boundaries of the duct. The ideal plane discontinuity is replaced by a series of progressively weaker shocks, with decreasing inter-shock distances in the core of the flow. Such a "pseudo-shock" (Figure 2) also occurs in supersonic diffusers [3], in the inlets of air-breathing engines and SCRAMjet engines, in supersonic compressors and high-pressure power plants [4] and even gas-dynamic lasers [5]. The shocks are accompanied by the corresponding fluctuations in flow parameters such as pressure, temperature and Mach number. The fluctuations continue until the core flow has been decelerated to sonic conditions. Thereafter the flow parameters change monotonically. The fluctuations are damped out at distances closer to the solid duct walls. Wall pressure measurements therefore exhibit a gradual rise instead of an abrupt one. The pseudo-shock can be divided into two parts: an upstream "shock" region and a downstream "mixing" region (Figure 3) [4]. Each of these regions can be further divided into two parts: core flow and boundary layer flow [11].

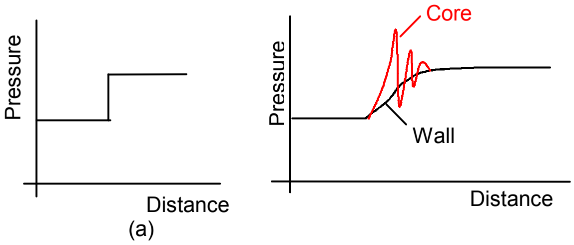

Figure 2 Single and Pseudo-Shocks

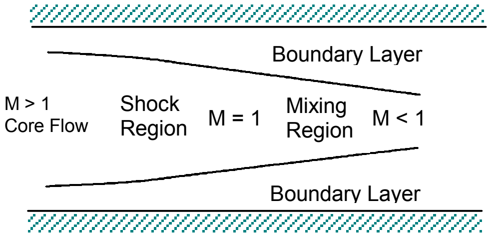

Figure 3 Pseudo-Shock Regions

Many experimental investigations of pseudo-shocks in clean air have been reported (e.g. [7-14]). Analytical investigations have been few and inadequate (e.g. [4, 11]). Previously established analytical models of the pseudo-shock in clean gas (the "Shockless", "Diffusion", and "Shock-Reflection" models) have both merits and shortcomings [1]. The "Modified-Fanno" model [1, 15] combines tested features of these models with a novel idea, resulting in a conceptually simple analytical model which can be easily extended to pseudo-shocks in dilute gas-particle suspensions.

“MODIFIED-FANNO” MODEL

Shock Region Flow and Fanno Flow

The overall characteristics of the flow in the shock region of the pseudo-shock in a duct are:

(1) Constant mass flux;
(2) Rigid and adiabatic duct walls;
(3) Flow tends towards the sonic condition.

It is observed that these characteristics are exactly the same as those of the well-known “Fanno” flow. In a Fanno flow, wall friction is solely responsible for changes in flow parameters in the downstream direction. The above observation, however, prompts the question: “Would it be possible to describe the flow in the shock region as a “modified” Fanno flow?” In this version, not only wall friction, but also progressively weaker shocks in the core would bring about changes in flow parameters. With this modification, it is possible to represent the successive states attained by the fluid in the shock region by points on the Fanno line, as shown in Figure 4 [1, 15].

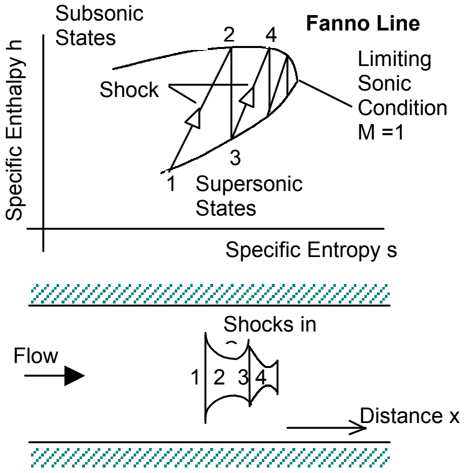

Figure 4. Pseudo-Shock as “Modified-Fanno” Flow

The physical mechanism of the pseudo-shock can be described as follows:

(1) At state ‘1’, the first (normal) shock appears in the core, provided the “blockage” due to boundary-layer growth is sufficient [7]. The core flow is decelerated to the corresponding subsonic state ‘2’.
(2) Due to the severe adverse pressure gradient created by the first shock, boundary-layer separation occurs which results in a converging-diverging nozzle-like flow in the core [13]. The subsonic flow at state ‘2’ is again accelerated to the next supersonic state ‘3’.
(3) The above steps are repeated until the core flow has been decelerated to sonic conditions, after which no shock formation is possible.

Inter-Shock Distances

The shocks in the core are progressively weaker. Consequently, the rise in specific entropy ‘s’ associated with each shock is smaller than that associated with the preceding shock. It can be assumed that the core flow between shocks is isentropic. This allows the following approximation for the flow between, for example, states ‘1’ and ‘3’:

$$\begin{aligned}\Delta s_{1-3} &= \Delta s_{1-2} + \Delta s_{2-3} \approx \Delta s_{1-2} \\ &= Entropy\ Rise\ associated\ with\ 1st\ Shock\ only\end{aligned} \tag{1}$$

Then, the distance between sections ‘1’ and ‘3’ (between the first and second shock) can be estimated using the relation between entropy change ds and friction factor f:

$$ds = \frac{c_p(\gamma-1)M^2}{2}\frac{f}{D_H}dx \tag{2}$$

so that

$$\begin{aligned}\Delta s_{1-3} &= \int_1^3 ds = \int_1^3 \frac{c_p(\gamma-1)}{2}M^2\frac{f}{D_H}dx \\ &\approx \frac{c_p(\gamma-1)}{2}M_{av,1-3}{}^2 f_{av}\,\Delta\left(\frac{x}{D_H}\right)_{1-3}\end{aligned} \tag{3}$$

$$\Delta\left(\frac{x}{D_H}\right)_{1-3} = \frac{\Delta s_{1-3}}{\frac{c_p(\gamma-1)}{2}M_{av,1-3}{}^2 f_{av,core}} \tag{4}$$

The distances between each pair of successive shocks can be calculated in the same way, provided an estimate is available for the core friction factor $f_{av,core}$ [1, 15]. Such an estimate can be obtained from a Second-Law analysis of the shock region. This analysis also represents an extension of the “Diffusion” model [11].

Second-Law Analysis

Following the nomenclature in [16], the law of entropy production [15] can be written for the shock region as:

$$\dot{S}_{gen} = \frac{\partial S}{\partial t} - \frac{\dot{Q}}{T} + (\dot{m}s)_{end} - (\dot{m}s)_{start} \tag{5}$$

Under the assumption of steady and adiabatic flow,

$$\partial S/\partial t = 0;\ \dot{Q}/T = 0 \tag{6}$$

so that

$$\dot{S}_{gen} = (\dot{m}s)_{end} - (\dot{m}s)_{start} \tag{7}$$

Since only changes in entropy are significant, s_{start} may be assumed to be zero. Hence,

$$\begin{aligned}\dot{S}_{gen} &= \dot{m}_{total}\, s_{gen} = (\dot{m}s)_{end} \\ &= (\dot{m}s)_{end,core} + (\dot{m}s)_{end,boundary\ layer}\end{aligned} \tag{8}$$

(The steady flow assumption can be relaxed to include unsteady oscillations of the pseudo-shock [e.g. 17] without destroying the basic argument in this model.)

In terms of the ratio of mass flow rates [11]

$$\mu = \frac{\dot{m}_{boundary\ layer}}{\dot{m}_{core} + \dot{m}_{boundary\ layer}} \tag{9}$$

the entropy generated is given by:

$$s_{gen} = (1-\mu)_2\, s_{2,core} + \mu_2\, s_{2,boundary\,layer} \tag{10}$$

In terms of frictional dissipation factors f_{core} and $f_{boundary\ layer}$,

$$\begin{aligned}s_{gen} &= (1-\mu)_{end}\, f_{core}\int_1^3 \frac{c_p(\gamma-1)}{2}M_{core}{}^2\frac{dx}{D_H} \\ &+\mu_{end}\ f_{boundary\,layer}\int_1^3 \frac{c_p(\gamma-1)}{2}M_{boundary\,layer}{}^2\frac{dx}{D_H}\end{aligned} \tag{11}$$

This yields the (average) core friction factor:

$$f_{core} = \frac{s_{gen} - \mu_2 \int_1^3 c_p \frac{\gamma - 1}{2} M_{boundary\,layer}{}^2 f_{boundary\,layer}\, dx}{(1 - \mu_2)\int_1^3 c_p \frac{\gamma - 1}{2} M_{core}{}^2\, dx} \tag{12}$$

Finally, an estimate of s_{gen} can be obtained from the well known "Integrated Friction Factor" value: for a supersonic flow at an initial Mach number M to be driven to the sonic condition by frictional dissipative effects in a constant mass flux flow (i.e., a Fanno flow)[18]:

$$f_{\text{int}} \frac{L^*}{D} = \frac{1 - M^2}{\gamma M^2} + \frac{\gamma + 1}{2\gamma} \ln \frac{(\gamma + 1)M^2}{2 + (\gamma - 1)M^2} \tag{13}$$

where L* is the duct length required, and D its diameter. f_{int} is the "integrated friction factor", which can be looked upon as a weighted average of f_{core} and $f_{boundary\ layer}$. From this calculated value of core friction factor and upstream conditions, it is possible to compare analytical predictions with experimental results.

MODEL VALIDATION

Axisymmetric Flow

Figures 5 and 6 show a comparison between experimental results [7] and predictions using the modified-Fanno model.

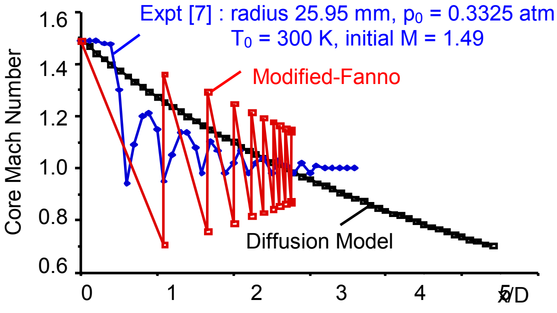

Figure 5 Core Mach Number Comparison

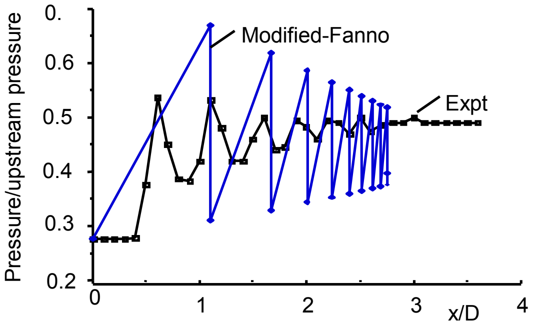

Figure 6 Core Pressure Comparison

Planar Flow

Comparison with Diffusion Model

Preliminary investigation of flow in a supersonic injection feeder [1] used a 30 mm width rectangular cross section duct with glass side walls is used. An "Air-only" test without the injection tube makes it possible to test the flow for evidence of pseudo-shocks in clean air. Wall pressure measurements allow comparison of between experiment and the prediction of the "Diffusion" model [11]. Figure 7 shows a typical set of wall pressure measurements.

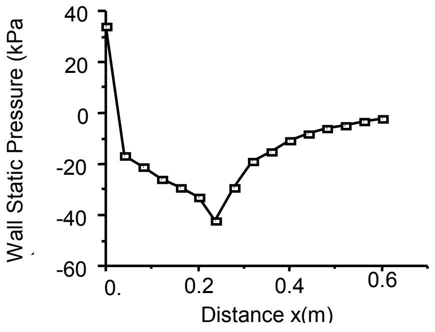

Figure 7 Air-Only Flow in Supersonic Injection Feeder

For a stagnation chamber pressure of 34 kPa (gauge), the minimum duct pressure attained is –42 kPa (gauge), corresponding to a Mach number of approximately 1.314. Subsequent pressure recovery to ambient conditions at the downstream end is seen to be gradual, not abrupt, thus indicating the presence of a pseudo-shock. Figure 8 shows a comparison between experiment and prediction by the Diffusion model. It is seen that at least for this moderately supersonic upstream Mach number, the Diffusion model predicts the pressure rise in the upstream part of the compression region with reasonable accuracy.

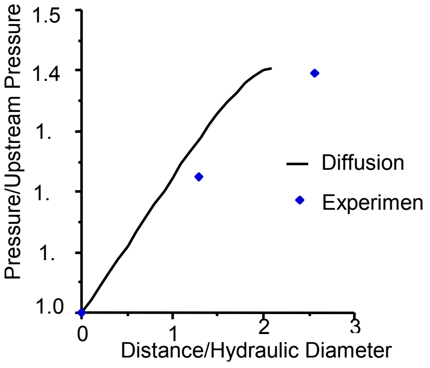

Figure 8 Wall Pressure Comparison

Comparison with Modified-Fanno Model

The shadowgraph technique [2] enables visualisation of the air-only flow in the core of the supersonic injection feeder duct. Figure 9 presents visual evidence of the existence of a series of shocks in the core.

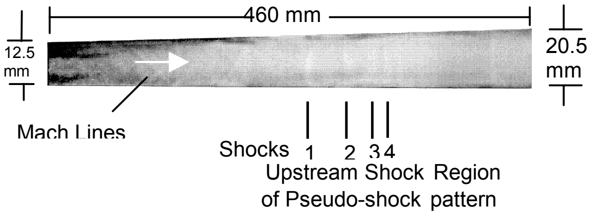

Figure 9 Shadowgraph of Pseudo-Shock

Mach lines emanate from the rough wall in the upstream part of the flow. They are clearer than the normal shocks comprising the pseudo-shock train, due to oscillations of the pseudo-shock about a mean position [17]. Existence of at least three or four shocks is evident from the bright lines in the shadowgraph. Inter-shock distances decrease in the downstream direction. The shocks are also progressively shorter, indicating gradual reduction in core area downstream of the initial shock. Figure 10 shows a comparison between core Mach number as predicted by the Modified-Fanno model, along with approximate locations of the shocks as revealed by the shadowgraph.

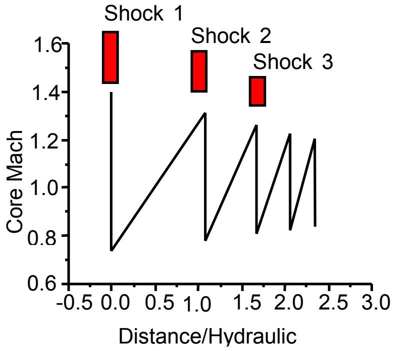

Figure 10 Core Mach Number (1)

Figure 11 shows a comparison between Mach number variation as predicted by the Diffusion model and the Modified-Fanno model.

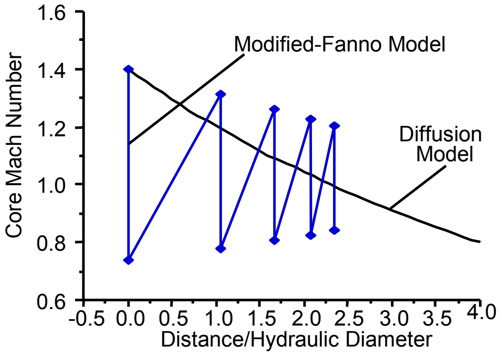

Figure 11 Core Mach Number (2)

The Diffusion model does not predict any shocks in the core of the flow, only an overall decrease in core Mach number. It is interesting to note that the Diffusion model predicts a total pseudo-shock length of about 4.6 duct diameters, out of which the upstream shock region occupies about 2.4 diameters. At this point, the Diffusion model predicts a core Mach number of almost exactly 1. This appears to confirm the reported finding [4] that the pseudo-shock can be divided into two distinct regions, the upstream shock region and the downstream mixing region. It is also interesting that the Modified-Fanno model, which is based on a one-dimensional analysis, can predict shocks in the core with reasonable accuracy even for a planar flow. The fourth or fifth shock predicted by the Modified-Fanno model is probably the "limiting" shock. Figure 12 shows a comparison between pressure variations as predicted by the Diffusion model, the Modified-Fanno model, and wall pressure measurements.

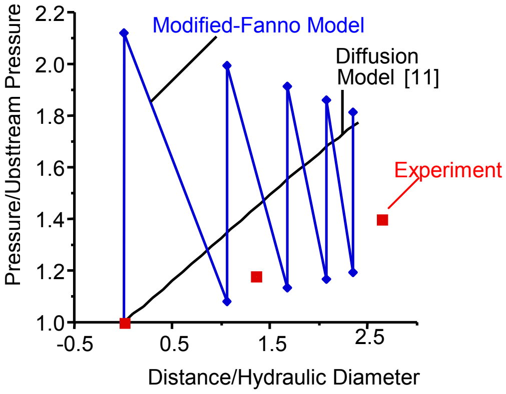

Figure 12 Pressure in Shock Region

PSEUDO-SHOCK IN DILUTE SUSPENSIONS

The simplest way to extend the above models to pseudo-shock in a gas-particle suspension is to assume that the suspension is dilute, with the solid particles occupying only a small fraction of the total volume. Under this assumption, the suspension behaves like a quasi-ideal gas whose properties can be expressed as functions of the solids volume fraction [e.g. 1, 19]. It is observed that the presence of solid particles greatly reduces the speed of sound in a gas-particle suspension, compared to that in clean gas. This implies that compressibility effects are heightened in suspensions, since even for relatively low velocities, the Mach number may not necessarily be small. This observation may have important implications for the investigation of pneumatic conveying of suspensions [19].

Extension of Diffusion Model

The length "l" of a pseudo-shock normalised with respect to duct (hydraulic) diameter "D_H" is [11]:

$$l=\frac{L}{D_H}=\frac{1}{c}\ln\left(\frac{\omega_1^{2}}{\omega^{*2}}\right) \tag{14}$$

where "c" is an experimentally determined constant, ω_1 the non-dimensional velocity (Crocco number) just upstream of the pseudo-shock, and ω * is a function of the isentropic exponent γ:

$$\omega_1=\frac{u_1}{\sqrt{2c_pT_0}}=M_1\sqrt{\frac{\gamma R}{2c_p}}\frac{1}{\sqrt{1+\frac{\gamma-1}{2}M^2}} \tag{15}$$

$$\omega^*=\frac{\gamma-1}{\gamma+1}$$

Since the constant-volume specific heat c_p, gas constant R and isentropic exponent γ can all be expressed as functions of particle loading, the parameters ω_1 and ω^* are also functions of particle loading, and so is the overall structure of a pseudo-shock in a suspension. It can be shown [1] that for the same upstream Mach number, a pseudo-shock in a suspension is longer than that in clean gas.

Extension of Modified-Fanno Model

Figure 13 shows a simple method of extending the modified-Fanno model to dilute gas-particle suspensions.

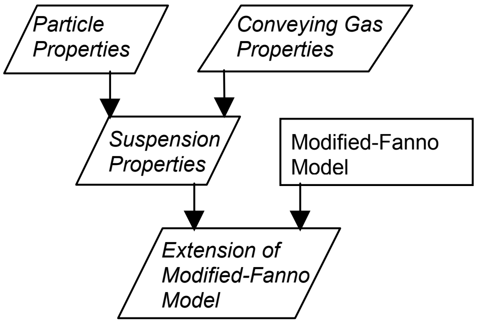

Figure 13 Simple Extension of Modified-Fanno Model

Figures 14 and 15 show a comparison between core Mach number variation and core pressure variation between comparable flows (same upstream Mach number) of clean gas and dilute suspension. At present, it is not possible to compare these predictions with experimental findings.

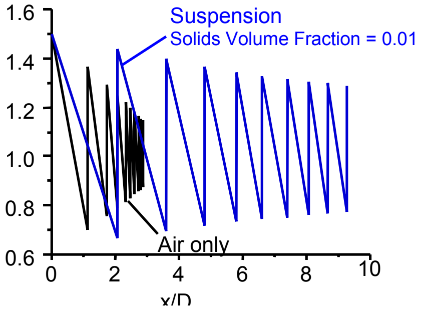

Figure 14 Core Mach Number (Suspension, Clean Gas)

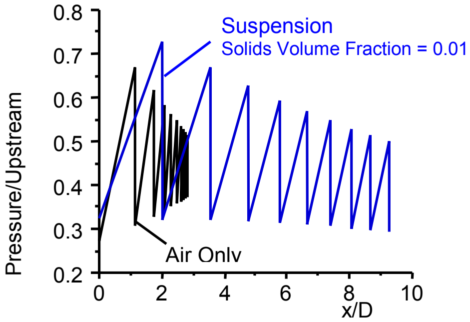

Figure 15 Core Pressure (Suspension, Clean Gas)

CFD SIMULATIONS

Cause of Pseudo-Shock

It is reported that severe shock-wave-boundary layer interaction is the sole cause of the pseudo-shock. Experimentally, this can be verified by sucking away the boundary layer through slots in the duct walls, which leaves a single shock in place of a pseudo-shock [3]. Alternatively, CFD simulations with the same mesh and inlet and exit boundary conditions can be conducted in two different ways: (1) without the "wall" condition, which eliminates shear layers growing along the solid duct walls, and (2) with the more realistic wall condition. In the former, there is an abrupt change in flow velocity (and other parameters) across the single normal shock (Fig. 16). The presence of shear layers on the walls in the latter case (Fig. 17) results in the alternately accelerating and decelerating flow in the core, a feature of the pseudo-shock. It is interesting to compare Figure 17 with Figure 3, which is the basis for the Modified-Fanno model.

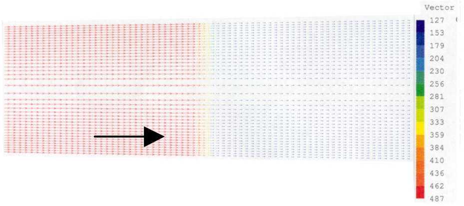

Figure 16 Without "Wall" Condition (PHOENICS)

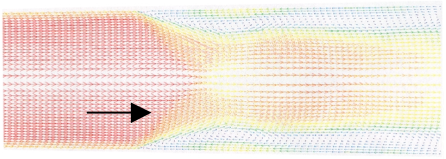

Figure 17 With "Wall" Condition (PHOENICS)

Pseudo-Shock in Clean Gas

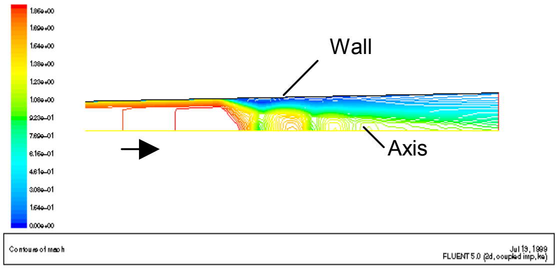

Figure 18 Mach Number Contours (Fluent 5.0.4)

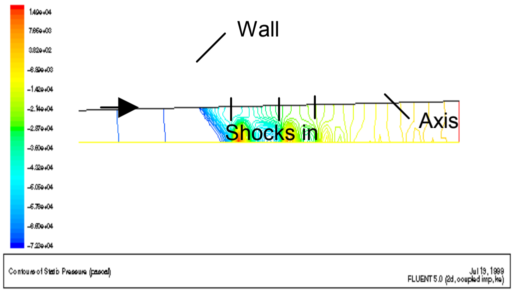

Figure 19 Pressure Contours (Fluent 5.0.4)

Figures 18 and 19 show results of a 2D simulation (Fluent 5.0.4, implicit coupled scheme) of the quasi-2D air flow in the rectangular cross section duct of the supersonic injection feeder for an upstream Mach number of about 1.8. Even with a relatively coarse mesh (150 x 15 cells) covering the top half of the symmetrical computational domain, the multiple shocks in the core are clearly seen. It is then possible to compare the variation of flow parameters on the axis with predictions by the modified-Fanno model for the same upstream Mach number (Figures 20 and 21).

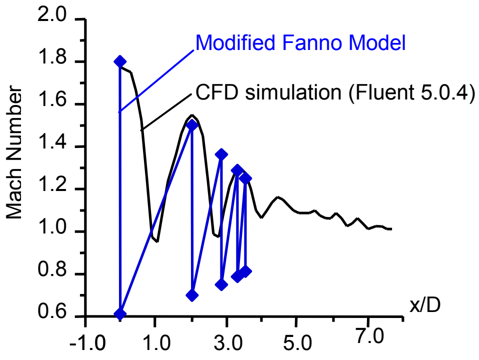

Figure 20 Core Mach Number

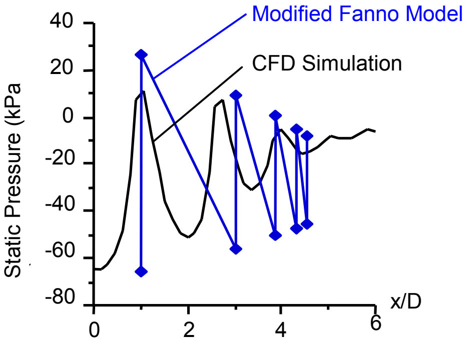

Figure 21 Core Pressure

Pseudo-Shock in Dilute Suspension

The suspension is treated as a quasi-ideal gas with modified properties, which are directly input. At present, the results of CFD simulation (Fluent 5.0.4) indicate shock formation at very low flow velocity (as low as 100 m/s), which would be a low subsonic speed in clean gas (air). This seems to reveal a heightened compressibility effect in the suspension, as predicted theoretically. The results also seem to indicate that the overall length of the pseudo-shock is increased in the suspension. More reliable CFD results will require additional input in the form of extended turbulence models. Experimental verification using the shadowgraph technique seems impossible, because the particles render the stream opaque to the passage of light through it.

CONCLUSION

The Modified-Fanno model seems to be able to predict all the parametric trends observed in pseudo-shocks in clean gas, in both axisymmetric and planar flows. Further investigation of the core friction factor parameter (e.g. dependence on Mach number) is needed. Experimental investigation of pseudo-shocks in gas-particle suspensions is needed, although CFD simulations do suggest that compressibility effects are heightened in suspensions.

ACKNOWLEDGMENT

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REFERENCES

1. GODBOLE, A.R., (1998), "Shock-Assisted Pneumatic Injection Technology", Ph.D. Thesis, University of Wollongong, Australia.
2. GODBOLE, A.R., et al (1998), "Supersonic Injection Feeder for Pneumatic Conveying", Proc. 6th Int Conf. on Bulk Materials Handling, Storage and Transportation, Wollongong, Australia.
3. SHAPIRO, A. H., (1953), "The Dynamics and Thermodynamics of Compressible Fluid Flow", Vols. 1, 2, The Ronald Press Co., New York.
4. YAMANE, R., et al, (1970), "A Simple Model of the Pseudo-Shock Mechanism", Bulletin of the Tokyo Inst. of Technology, No. 100.
5. YAMANE, R., et al, (1995), "Numerical Simulation of Pseudo-Shocks in Straight Channels", JSME Int. J. Series B, 38 (4)
6. YAMANE, R., (1996), Private Communication
7. OM, D., CHILDS, M.E., (1985), "Multiple Transonic Shock Wave/Turbulent Boundary Layer Interaction in a Circular Duct", AIAA J., 23(10), Oct., p 1506-1511.
8. OM, D., et al, (1985), "Transonic Shock Wave/Turbulent Boundary Layer Interaction in a Circular Duct", AIAA J., 23(5), May, p 707-714.
9. TAMAKI, T., et al., (1970), "A Study of Pseudo-Shock (1st Report, λ-type Pseudo-Shock), Bulletin of the JSME, 13(55), p 51-58.
10. TAMAKI, T., et al., (1970), "A Study of Pseudo-Shock (2nd Report, X-type Pseudo-Shock), Bulletin of the JSME, 14(74), p 807-817.
11. IKUI, T., et al., (1974), "The Mechanism of Pseudo-Shock Waves", Bulletin of the JSME, 17(108), June, p 731-739.
12. CARROLL, B. F., DUTTON, J. C., (1990), "Characteristics of Multiple Shock wave /turbulent boundary-layer interactions in rectangular ducts", J Propulsion, 6(2), Mar-Apr, p 186-193
13. CARROLL, B. F., DUTTON, J. C., (1992), "Turbulence phenomena in a multiple normal shock wave/turbulent boundary layer interaction", AIAA J, 30(1), Jan, p 43-48
14. CARROLL, B. F., DUTTON, J. C., (1992), "Multiple normal shock wave/turbulent boundary-layer interactions", J Prop and Power, 8(2), 1992, p. 441-448
15. GODBOLE, A. R., et al, (1997), "The Pseudo-Normal Shock as 'Modified-Fanno' Flow", Proceedings of IASTED MSO'97, Singapore, p. 91-95
16. BEJAN, A., (1982), "Entropy Generation through Heat and Fluid Flow", John Wiley and Sons.
17. IKUI, T., MATSUO, K., (1974), "Oscillation phenomena of pseudo-shock waves", Bulletin of the JSME, 17(112), Oct, p. 1278-85
18. WHITE, F. M., (1994), "Fluid Mechanics", Third Edition, McGraw-Hill.
19. WALLIS, G.B., (1969), "Suspensions of Particles in Fluids", One-Dimensional Two-Phase Flow, McGraw-Hill
20. GODBOLE, A.R., WYPYCH, P.W., (1998), "Application of Gas-Dynamic Principles to Pneumatic Conveying of Suspensions", Proc. 6th Int Conf. on Bulk Materials Handling, Storage and Transportation, Wollongong, Australia.