

## VALIDATION OF A CFD MODEL OF A THREE-DIMENSIONAL TUBE-IN-TUBE HEAT EXCHANGER

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### ABSTRACT

Computational Fluid Dynamics (CFD) was developed to model many flow types and in addition, can be used to solve heat transfer problems. A tube-in-tube exchanger, with hot water (single phase) flowing in the inner tube and cold water in the annulus was investigated. The heat exchanger was numerically modelled in three- dimensions in CFD. The heat transfer coefficients and the friction factors were determined with CFD and compared to established correlations. The results showed reasonable agreement with empirical correlations, while the trends were similar. When compared with experimental data the CFD model's results showed good agreement. The second part of the study investigated the CFD's ability to model a prototype configuration of a tube-in-tube exchanger. This ability will greatly reduce cost and time when developing a new heat exchanger. The numerical data was compared with analytical predictions and experimental results. Recommendations were made on CFD's value as a tool to characterise an exchanger.

### NOMENCLATURE

$a$	annular diameter ratio
$A$	area
$C$	circumference
$c_p$	specific heat
$d_1$	diameter of outer wall of inner tube
$d_2$	diameter of inner wall of outer tube
$d_i$	inner diameter of inner tube
$d_h$	hydraulic diameter
$f$	friction factor
$h$	convective heat transfer coefficient
$k$	thermal conductivity
$k$	turbulent kinetic energy
$L$	length of heat exchanger
$\dot{m}$	mass flow rate
$n$	number of times fractal is applied
Nu	Nusselt number
$\Delta p$	pressure drop
Pr	Prandtl number
$q$	heat transfer
$q_0$	benchmark heat transfer
Re	Reynolds number
$\Delta T_{LMTD}$	log-mean temperature difference
$T$	temperature
$T_\infty$	free stream temperature
$U$	overall heat transfer coefficient
$v$	flow velocity
$x_o$	original square length

$\alpha$	variable
$\varepsilon$	dissipation rate
$\gamma$	corrective function
$\mu$	viscosity
$\rho$	density

### Subscripts

$i$	inner
$w$	wall

### INTRODUCTION

CFD has been applied to solve many thermodynamics and heat transfer problems. It includes investigations into the performance of heat exchangers. One such application was studied by Rustum and Soliman, (1990) who investigated an internally finned tube. In another study, plate heat exchangers were modelled in CFD and the hydrodynamics were studied. The paper showed that CFD could assist in the optimal design of plate exchangers (Grijpspeerd *et al.*, 2003). CFD was also used to characterise a cardioplegia heat exchanger (Van Driel, 2000). The predicted CFD results were validated with experimental data. Other types of exchangers that were modelled with CFD, include a vertical mantle exchanger. Reliable results were found when compared with outdoor measurements (Shah, 2000). One of the advantages of CFD is that CFD can be applied to study trends and properties (Book, 1981). Therefore, an exchanger's configuration and material can easily be modified to investigate design changes.

The purpose of this study was to determine the accuracy of a three-dimensional CFD model of a simple tube-in-tube exchanger when compared with empirical correlations. The CFD results were also compared with values obtained from an independent experimental study. One of the main applications of numerical investigations is to determine the characteristics of a prototype exchanger to establish its suitability for a particular application. This paper will study the CFD's ability to model an uncharacterised prototype heat exchanger. Since there were no symmetrical planes the exchanger was configured three-dimensionally. The CFD results were then compared with experimental values.

### CFD MODELLING OF A SIMPLE TUBE-IN-TUBE EXCHANGER

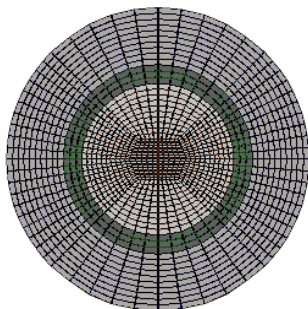
To characterise an exchanger, the heat transfer and pressure drop characteristics were needed and this was achieved by establishing the dimensionless formulas for the Nusselt number and friction factor. The heat exchanger

to be modelled numerically consisted of two tubes, one within the other. The inner tube's inside diameter was 8 mm, with an outer diameter of 10 mm. The inside diameter of the outer tube was 16 mm. Since steady state conditions were assumed, the length of the heat exchanger was limited to 50 mm. The tube material was copper with a conductivity of 386 W/mK.

Hot water flowed in the inner tube, with cold water flowing in the annulus in the opposite direction. The inner flow velocities were taken as 0.5, 0.8, 1.4 and 2 m/s. The velocities for the annulus were 1, 2.8, 4.9 and 7 m/s. The inner tube inlet temperature was assumed to be 82°C and the annulus inlet temperature was taken as 10°C.

The simulation package used is called Star-CD and makes use of the finite volume method. CFD solved for temperature, pressure and flow velocity at every cell. In order to do this it utilised the differential forms of the Navier-Stokes equations to model fluid flow and the  $k-\epsilon$  model for standard turbulent flow. Heat transfer was modelled through the enthalpy conservation equation. The outer tube was not modelled; its presence was accounted for by the introduction of an adiabatic boundary at the top of the outer fluid. The inlet and outlet boundaries were also defined which characterised the flow velocities. The regions without boundaries were considered heat transfer conduction boundaries. On average the CFD model converged in 116 minutes (CPU time) and in 110 iterations.

The amount of cells used was 237 000; 120 000 cells for the modelling of the inner fluid, 27 000 cells for the copper tube and 90 000 cells for the annulus fluid. The inner tube of the exchanger as well as the inner and annulus flow was modelled with three-dimensional cells as shown in figure 1.



**Figure 1:** Front view of the CFD exchanger model.

#### Calculation of the Nusselt Number

After the CFD model had converged, the temperature and pressure at the inlets and outlets were determined so that the heat transfer, Nusselt number and friction factor could be calculated. Since the size and the volume of the cells were dissimilar, a larger weighted value was allocated to a larger cell compared to a smaller cell. All the temperature and pressure drop values were determined by mass-averaging them over a certain area; thus improving the results. The heat transfer was determined by the following equation:

$$q = \dot{m}c_p(T_{out} - T_{in}) \quad (1)$$

A heat balance was established by ensuring a small error (average error was 5.1%) between the heat transfer from

the inner tube and the heat transfer to the annulus. The heat transfer coefficient was calculated by using the heat transfer value obtained and the equation below:

$$h = \frac{q}{A(T_w - T_\infty)} \quad (2)$$

The Nusselt number is a dimensionless number that incorporates the heat transfer coefficient in its equation and can be calculated as follows:

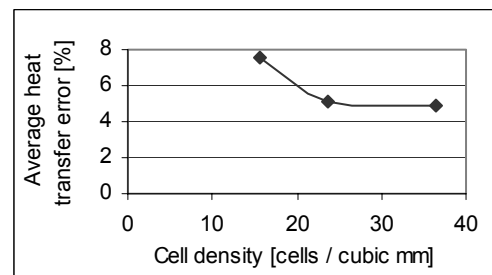
$$Nu = \frac{hd_h}{k} \quad (3)$$

The dimensionless number that incorporates the pressure drop through the exchanger is the friction factor. The friction factors were calculated from the pressure drop found from the CFD results, utilizing the following equation (Holman, 1992):

$$f = \frac{2d_h\Delta p}{\rho Lv^2} \quad (4)$$

#### Sensitivity analysis

A sensitivity analysis was done to ensure that the optimal amount of cells was used. Three different cell densities (amount of cells divided by the volume) were modelled and the results compared with each other. The model with the least amount of cells had a density of 15.518 cells/mm<sup>3</sup>, followed by a medium cell density of 23.575 cells/mm<sup>3</sup> and a high cell density of 36.526 cells/mm<sup>3</sup>. To compare the three models, the error in heat transfer between the inner and annulus flow was used as the main criteria. Figure 2 shows the relationship between the error and the cell density.



**Figure 2:** Sensitivity analysis

From the figure it can be seen that from a density larger than 20 cells/mm<sup>3</sup> the error became a minimum. The medium cell density model would be used since it produced good results and converged in less time and iterations than the high cell density model. The Nusselt number and friction factor results obtained from the simulated CFD models will be discussed in the following section.

## RESULTS OF THE CFD MODEL

#### Calculation of the Nusselt Number

Heat transfer values were obtained and used to find the heat transfer coefficient and the Nusselt number. To determine its accuracy the CFD values were compared to the Dittus-Boelter correlation:

$$Nu = 0.023 Re^{0.8} Pr^n \quad (5)$$

With  $n = 0.4$  for heating and  $n = 0.3$  for cooling.

This correlation was used because of its simplicity. When the results of the above correlation were compared with the CFD results, the average error for the inner Nusselt

numbers was 18.8% while for the annulus flow the error was 4.6%. The CFD and Dittus-Boelter Nusselt numbers are shown in figures 3 and 4.

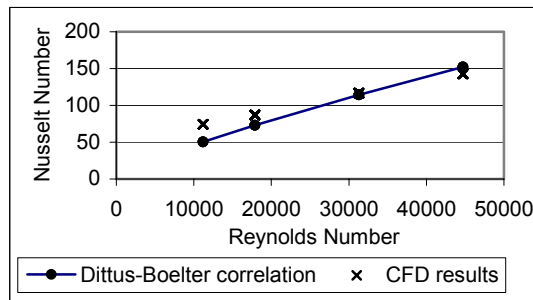


Figure 3: Inner Nusselt numbers.

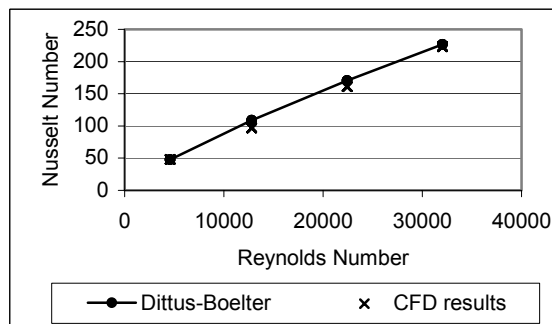


Figure 4: Annulus Nusselt number.

The above figures show that CFD models the Nusselt number closely and that the trends were also very similar. Unfortunately attempts to increase the Nusselt number usually results in an increase in pressure drop. This factor was investigated in the next section.

#### Calculation of the Friction Factor

The friction factors were calculated from the pressure drops using equation (4). The CFD values were compared to the correlation values below (Holman, 1992) to determine the accuracy of the CFD results:

$$f = (1.82 \log_{10} Re - 1.64)^{-2} \quad (6)$$

For the inner flow the average error was 39.7%, while the error for the annulus flow was 35.0%. The CFD friction trends were similar (though higher) than the correlation trends (refer to figure 5).

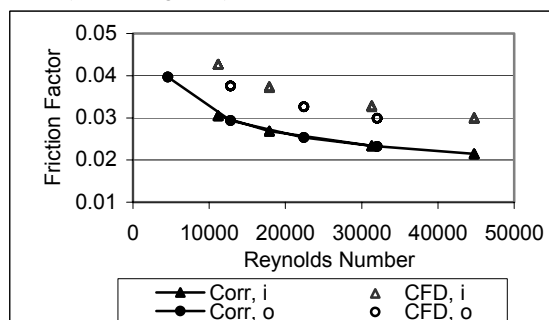


Figure 5: Friction factor comparison.

Many factors could influence the friction results, such as the smoothness of the tubing.

The CFD model was not only compared to empirical correlations, but also to experimental results. The experimental set-up and results along with the comparison will be discussed in the following section.

## EXPERIMENTAL STUDY

The experimental study was done independently of this investigation. The purpose of the study was to determine experimentally the Nusselt number correlation for the annulus of a tube-in-tube exchanger. The correlation also took the annular diameter ratio into account. This ratio is defined as the ratio of the outer tube's inner diameter ( $d_2$ ) to the inner tube's outer diameter ( $d_1$ ):

$$a = \frac{d_2}{d_1} \quad (7)$$

The experimental set-up to determine this correlation was similar to the CFD's set-up.

#### Experimental Set-Up

The heat exchangers were manufactured from hard drawn refrigeration copper tubing, with a length of approximately 6 m. Eight heat exchangers were tested with the following dimension ranges:

$$5.3 \leq d_i \leq 17.3 \text{ mm}$$

$$6.35 \leq d_o \leq 19.5 \text{ mm}$$

$$11.15 \leq d_2 \leq 32.0 \text{ mm}$$

The inner tube was positioned concentric inside the outer tube by means of spacer pins. Hot water flowed in the inner tube, while cold water flowed in the opposite direction in the annulus. The inner flow rate was kept constant while the annulus flow was varied. This process was then repeated by choosing a new inner flow rate. The Reynolds number varied between 4000 and 30000 in the annulus.

When steady state conditions were achieved the temperatures were measured with K-type thermocouples. The volumetric flow was measured by semi-rotary circular piston type flow meters installed at the exits. The temperature and flow measurements were used to determine the correlation.

#### Experimental Results

The modified Wilson method (Briggs and Young, 1969) was used to find the annulus Nusselt number correlation. The ratio of the diameters was incorporated afterwards. The resultant Nusselt number correlation for turbulent flow in the annulus was found:

$$Nu = \frac{0.003a^{1.86}}{\gamma} Re^\alpha Pr^{\frac{1}{3}} \left( \frac{\mu}{\mu_w} \right)^{0.14} \quad (8)$$

where

$$\gamma = 0.063a^3 - 0.674a^2 + 2.225a - 1.157 \quad (9)$$

$$\alpha = 1.013e^{-0.067a} \quad (10)$$

The above correlation has an accuracy of 3% for a Reynolds number range of 4000 and 30000 for the annulus with  $a = 1.7$  to 5.1 (Dirker, 2002). This correlation was used to compare the CFD values with.

#### Comparison with CFD Results

Figure 6 shows the comparison between the CFD model and the values for the correlation. The average error was 5.5% and the results compared well with the correlation. It can be concluded that the CFD software modelled a tube-

in-tube heat exchanger in three-dimensions accurately. The next investigation will study CFD's ability to configure a prototype exchanger.

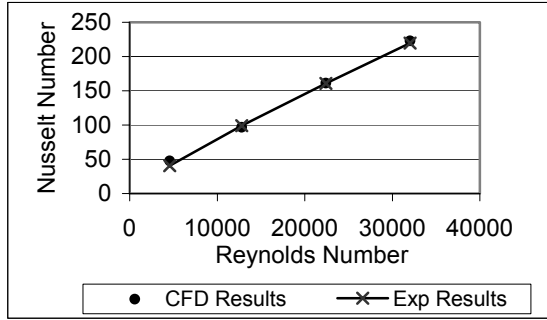


Figure 6: Annulus Nusselt numbers.

### CFD'S ABILITY TO MODEL A PROTOTYPE HEAT EXCHANGER

One of the applications of CFD is to model new designs of heat exchangers and to find their characteristics before investing in manufacture costs.

#### Fractal Heat Exchanger

A fractal heat exchanger is a tube-in-tube exchanger where the inner tube's configuration was modified with a fractal. Fractals are mathematical concepts that are generated with repeated application of certain rules and whose dimension is not an integer but a fraction (Peitgen *et al.*, 1993). Figure 7 shows the first couple of fractal iterations of the quadratic Koch island fractal.

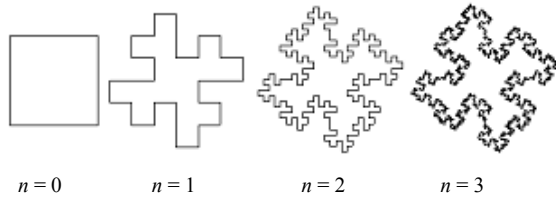


Figure 7: The Quadratic Koch island fractal.

The above figure shows the original square ( $n = 0$ ) on which the fractal was applied. The next three figures in figure 7 shows the square after the fractal was applied once ( $n = 1$ ), twice ( $n = 2$ ) and three times ( $n = 3$ ). The heat transfer, Nusselt number and friction factors were dependent on the number of times the fractal was applied. The fractal properties were derived first after which the fractal theory was applied to heat transfer.

The cross-sectional area stayed the same since for every square that was added the same amount was subtracted elsewhere. The heat transfer area (which is a function of the circumference) increased with every fractal application. The circumference doubled with every application of the fractal, because the length of the fractal applied was double the length of the section it was applied to:

$$C = 4(2^n x_0) \quad (11)$$

When this fractal was used on the inner tube of a tube-in-tube exchanger, the heat transfer area doubled with every application of the fractal if the overall heat transfer coefficient and the log-mean temperature difference remained constant. Through analytical techniques the increases in heat transfer, Nusselt number and friction

factors were estimated. The heat transfer was defined as (Holman, 1992):

$$q = UAAT_{LMTD} \quad (12)$$

When the area doubled with each fractal iteration, the heat transfer would double as well. The overall heat transfer coefficient is a function of, among other variables, the heat transfer coefficient, which can be written in terms of the Nusselt number. The Dittus-Boelter correlation (equation (5)) for the Nusselt number was used for the estimations, because of its simplicity. The correlation was a function of the Reynolds number, which is also dependent on the fractal iteration. The inner Reynolds numbers was determined as (Van der Vyver, 2003):

$$Re_i = \frac{\rho v_i d_{h,i}}{\mu} = \frac{\rho v_i d_i}{2^n \mu} \quad (13)$$

Inserting the above Reynolds equation into the Dittus-Boelter correlation resulted in the following:

$$Nu_i = 0.023 \cdot \left( \frac{\rho v_i d_i}{2^n \mu} \right)^{0.8} Pr^{0.3} \propto 2^{-0.8n} \quad (14)$$

The Nusselt number decreased with the application of the fractal. The same trend was found for the annulus flow. Incorporating the definition of the Nusselt number, the heat transfer coefficient's dependency on the fractal was found:

$$Nu = \frac{hd}{2^n k} \propto 2^{-n} h(n) \propto 2^{-0.8n} \quad (15)$$

$$\therefore h \propto 2^{0.2n} \quad (16)$$

The heat transfer coefficient increased with the fractal iteration. It can be concluded that the overall heat transfer coefficient was also dependent on the fractal iteration. The calculation of the estimated increase in the overall heat transfer coefficient was very lengthy. To simplify the estimation process, it was decided to neglect the influence of the overall heat transfer coefficient. It could then be concluded that the heat transfer at least doubled, stated mathematically as:

$$q_n \propto 2^n (q_0) \quad (17)$$

The above equations showed that the heat transfer increased with the fractal iteration. This increase was counter-acted by the increase in pressure drop and the resultant friction factor. The estimated friction was determined by Van der Vyver (2003) and is shown as:

$$f \propto 2^{0.311n} \quad (18)$$

The above analytical estimations were performed by assuming many simplifications. To validate these predictions, CFD was utilised to model the exchangers and to determine the increases over the benchmark equation.

#### CFD Model of a Fractal Heat Exchanger

Three different fractal heat exchangers, corresponding to  $n = 0, 1$  and  $2$  were modelled (refer to figure 8). The physical properties of the above models are summarised in table 1.

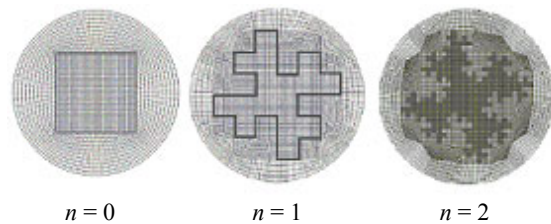


Figure 8: Three fractal exchangers CFD models.

	$n = 0$	$n = 1$	$n = 2$
Inner tube inside width ( $x_0$ ) [mm]	42	n/a	n/a
Inner tube thickness [mm]	1	1	n/a
Inner diameter of outer tube [mm]	90	90	90
Length of heat exchanger [mm]	125	250	125
Inner hydraulic diameter [mm]	42	20.2	10.5
Annulus hydraulic diameter [mm]	38.6	26.6	19.3
Total amount of cells	347225	349500	537040
Inner Reynolds numbers	9394 - 93944	4527 - 45272	9385 - 43995
Annulus Reynolds numbers	5004 - 50039	3450 - 34504	13235 - 44117

**Table 1:** Properties of the fractal heat exchangers.

The fluid properties for each of the fractal heat exchangers were the same and are presented next.

The inner inlet fluid temperature was taken as 82°C, while the inlet annulus fluid temperature was 10°C. The turbulence intensity for both fluids was 0.05. The tube material was taken as aluminium since the actual exchanger would be manufactured from aluminium. The material had a density of 2787 kg/m<sup>3</sup>, conductivity of 164 W/mK and a specific heat of 883 J/kgK.

For the second iteration ( $n = 2$ ) the aluminium tube was not modelled due to the limitations the CFD software imposed on the number of cells. It was modelled with two-dimensional cells called baffles. The baffles were defined as conduction baffles so that heat transfer across the baffle could take place.

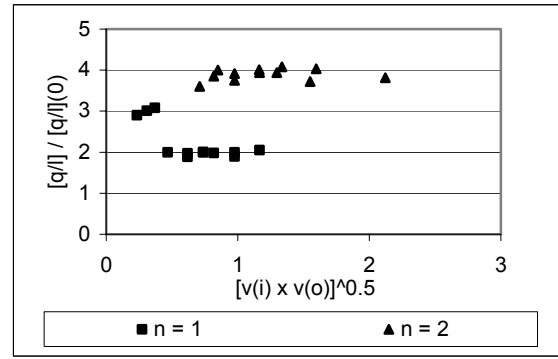
The CFD model was implemented in the same manner as the simple tube-in-tube exchanger. In addition the same method was used to acquire the results and to determine the resultant heat transfer, Nusselt numbers and friction factors.

### Results

The first model ( $n = 0$ ) converged, on average in 147 minutes (CPU time) and in 77 iterations. The next model ( $n = 1$ ) converged in 130 minutes (55 iterations) and the last model ( $n = 2$ ) converged in 388 minutes (91 iterations). The discrepancy between  $n = 1$  and the other models' convergence time was due to different lengths of heat exchangers modelled.

The heat balance errors for the three fractal exchangers were 5.3% ( $n = 0$ ), 4.4% ( $n = 1$ ) and 4.7% ( $n = 2$ ), which were sufficiently small. The heat transfer showed an increase over the benchmark exchanger ( $n = 0$ ) of 2.1 times ( $n = 1$ ) and 3.9 times ( $n = 2$ ). Refer to equation (19) and figure 9.

$$\text{Increase} = \frac{(q)_{1,2}}{(q)_0} \quad (19)$$



**Figure 9:** Increases over the benchmark exchanger.

This compared well with the predicted values of equation (17) of 2 ( $n = 1$ ) and 4 ( $n = 2$ ) times. From the heat transfer calculated the heat transfer coefficients and the Nusselt numbers were determined. The increases in the Nusselt number were calculated with respect to the velocity (since the velocities were different) by utilizing the following equation:

$$\text{Increase} = \frac{(Nu/v)_{1,2}}{(Nu/v)_0} \quad (20)$$

The average decreases for the Nusselt numbers were 0.65 ( $n = 1$ ) and 0.35 ( $n = 2$ ) times. These decreases were compared with the predicted decrease of equation (15) of 0.57 ( $n = 1$ ) times and 0.33 ( $n = 2$ ) times. The CFD results were in good agreement with the estimated results. The last estimation that was validated was the increase in friction factors. From equation (18) the estimated increases were 1.24 times ( $n = 1$ ) and 1.54 times ( $n = 2$ ). For the CFD results the increases were determined from the following equation:

$$\text{Increase} = \frac{\left(\frac{f}{v}\right)_{1,2}}{\left(\frac{f}{v}\right)_0} \quad (21)$$

The CFD results predicted decreases of 0.95 times ( $n = 1$ ) and 0.48 times ( $n = 2$ ). This estimation could not be validated with CFD. One possible reason for this discrepancy was that from figure 4 it was shown that the CFD resulted in large errors when compared with the friction empirical correlation. Thus, the CFD validated the heat transfer increases and the Nusselt number estimations. The analytical estimation of equations (14) and (17) were also validated experimentally.

### Comparison with Experimental Results

The aluminium fractal tube ( $n = 1$ ) was manufactured by means of extrusion. The dimensions are shown below:

Heat exchange length	3910 mm
Outside diameter of round tube	88.9 mm
Inside diameter of round tube	79.34 mm
Outer fractal length	13 mm
Inner fractal length	7 mm
Inner cross sectional area	1129 x 10 <sup>-6</sup> m <sup>2</sup>
Annulus cross sectional area	2855 x 10 <sup>-6</sup> m <sup>2</sup>
Inner hydraulic diameter	14.662 mm
Annulus hydraulic diameter	19.647 mm

**Table 2:** Physical measurements of the fractal heat exchanger.

The experimental set-up remained the same as the simple tube-in-tube exchanger. The temperature, flow rates and pressure drops were measured.

The experiments were done for a Reynolds number range of 2706 to 18325 for inner flow and 758 to 7440 for the annulus flow. Several experiments were performed and the heat transfer was calculated from equation (1). To determine the heat transfer increases the fractal exchanger was compared to the benchmark exchanger ( $n = 0$ ). This exchanger consisted of a simple square tube inside a round outer tube. The inner cross sectional area, length and inner diameter of the outer tube of the benchmark exchanger was the same as the experimental exchanger.

The heat transfer was calculated from equation (12), where the log-mean temperature difference was taken from the experimental investigation. The volumetric flow was taken to be the same as well. The values for the heat transfer coefficients were obtained from the Dittus-Boelter correlation, equation (5).

The results from the two exchangers were calculated and compared. The average increase in heat transfer was 2.46 times. This was higher than the anticipated double increase and was closer to the CFD increase of 2.1 times. Thus other factors such as the heat transfer coefficient could also be a function of the fractal iteration. The heat transfer coefficient was incorporated in the Nusselt number and the Nusselt number increases were considered next.

The difference between the  $n = 0$  and  $n = 1$  for the Nusselt values was determined below:

$$\text{Increase} = \frac{Nu_1}{Nu_0} \quad (22)$$

The average decrease in the Nusselt number was 0.66 times over the benchmark exchanger. This is very close to the value found for the CFD analysis of 0.65 times. The decreases in Nusselt number were less than the analytical estimation of 0.57 times.

For the inner flow, good correlation was found for the friction factors over a Reynolds number range of 7500 and 18000. The average error was  $\pm 10\%$ . The annulus friction factors were not validated with experimental results. There were differences in the position of the pressure measuring points between the CFD and actual experimental set-up. In addition the annulus flow bent at the inlet and outlet. Thus different results were expected and found and a comparison could not be made.

The experimental results show good agreement with the CFD calculations. Thus CFD can model a prototype heat exchanger accurately and can be used to determine the characteristics of a new design of exchanger. The two above studies resulted in some recommendations for modelling actual heat exchangers with CFD.

## RECOMMENDATIONS

The study provided many recommendations to model three-dimensional heat exchangers with CFD. The main recommendations are summarised below.

Before modelling a new type of exchanger it is important to model and implement a basic heat exchanger. The comparison between the CFD's results and experimental results will be an indication of the software's accuracy. It will also provide the researcher the opportunity to become familiar with the software.

Although the CFD model will be a simplification of the actual exchanger, it should model the experimental set-up as closely as possible. It should include hose attachments, changes in inlet and outlet diameters and changes to the direction of flow.

Where the software provides a summary of the results, the results should be checked. This recommendation is especially valid for symmetrical boundaries where the summary may state only half of the actual heat transfer.

## CONCLUSION

CFD accurately predicted heat transfer and Nusselt numbers for a three-dimensional simple tube-in-tube exchanger. Similarly CFD provided good agreement with analytical and experimental results for a prototype exchanger.

Where the experimental and CFD flows were similar, a good correlation for friction was found between the CFD and experimental results. It can be concluded that CFD is a valuable tool in heat exchanger design.

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