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A NEW MODEL FOR MHD INSTABILITIES IN ALUMINUM REDUCTION CELLS

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ABSTRACT

A new model for investigating the interfacial MHD instabilities in aluminium reduction cells in form of a wave equation based on Shallow Water approximation and linear stability analysis is proposed and the effect of different parameters of the cell on its instability phenomenon is considered. This work has two main features that are: (a) the terms of linear friction have been considered in Aluminium and Cryolite layers so that we can see the stabilizing effects of these terms in form of a damping term for interfacial wave Equation. (b) In our numerical method we have considered the effect of lateral boundaries on the stability of aluminium reduction cell. Results of this model can be used in designing new cells and control systems for aluminium reduction cells.

NOMENCLATURE

a	indices for aluminium
c	indices for cryolite
p	pressure
u	velocity
J0	*Unperturbed electric current density*
j	*Perturbed electric current density*
η	*Deviation of interface from steady state*
h_1	*Thickness of aluminium layer*
h_2	*Thickness of cryolite layer*
L_x, L_y	*Horizontal dimensions of cell*
κ	*Empirical linear friction coefficient*
g	*gravity acceleration*
σ	*Electrical conductivity*
ϕ	*Electrical potential*
B0	*Unperturbed magnetic field*
B	*Perturbation of magnetic field*
ρ	Density

INTRODUCTION

It is more than a century that Aluminium production around the world is done based on Hall-Heroult process. In this process Aluminium Oxide (Al_2O_3) is reduced to Aluminium in Electrolytic Reduction Cells. The schematic geometry of a typical cell and also a simplified model used for modelling is illustrated in Fig. 1.

Cell is a steel rectangular box that its inner surface is covered by a carbon lining and the layer of molten cryolite (Na_3AlF_6), in which powdered alumina is dissolved, is superimposed on the layer of molten aluminium that is produced during the reduction process. The aluminium and the carbon made walls of a rectangular bath act as a cathode. The carbon anodes are immersed into the cryolite.

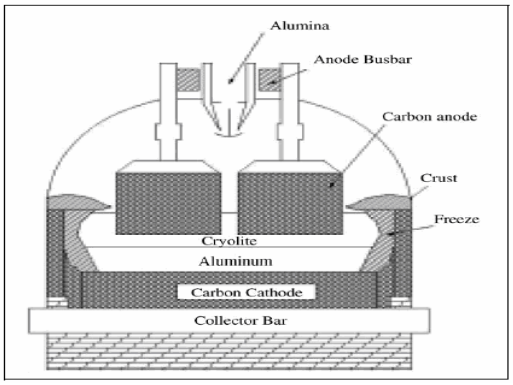

Figure 1.a: Schematic geometry of an aluminium reduction cell

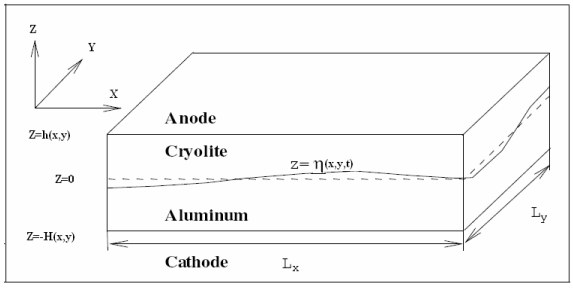

Figure 1.b: Simplified geometry used in modelling

Due to almost perfect immiscibility of molten aluminium and cryolite layers and their different densities (densities of aluminium and cryolite are $\rho_a = 2250kg/m^3$, $\rho_c = 2075kg/m^3$, respectively). There is a separating interface between two molten layers. The electrolytic reduction occurs at the interface under the action of strong electric current J_0 (the total current is between 100 and 500kA) which passes from the anode to the cathode. Strong magnetic field (up to $B_0 \approx 0.01T$) are induced by the currents that flow within the cells and in the conducting lines in the plant.

The process requires significant amount of electrical energy. The energy consumption by a modern cell is about 14kwh per kg of produced aluminium. A large portion of the electrical energy is wasted by ohmic heating in cryolite layer because of very low electrical conductivity in this layer $\sigma_{cryolite} = 200\Omega^{-1}m^{-1}$. The only way for minimization of these energy losses is to reduce the

distance between the bottom of the anode and the aluminium – cryolite interface (so called ACD or anode – cathode distance) to a level that is as small as possible. But, the reduction of ACD bellows a certain distance is impossible since it results in instability of the interface. It has hardly been possible in modern cells to reduce ACD to less than 4 cm.

In the view of energy efficiency in primary aluminium production interface instability is a very important problem and has long been the subject of investigation (see, e.g., sele, 1977; Urata, 1985; Moreau and Evans, 1984; Moreau and Ziegler, 1988). In these works the instability was introduced as a complex process involving interactions between the interface movements, electric currents, melt flows and magnetic fields. Significant progress in the theoretical modelling of the instability was achieved in the works by Bojarevics and Romerio (1994), Sneyd and Wang (1994), and Davidson and Lindsay (1998). As a result of these works it has been found that long wavelength interfacial waves could be amplified by the electromagnetic Lorenz forces arising from interaction between the horizontal current perturbations in the aluminium layer and the external imposed magnetic field and therefore this instability has MHD origin. A new approach for modelling of instability was proposed by Zikanov et al (2000). In that model the flows of aluminum and cryolite and the interface deformation were simulated numerically as coupled part of a fully non-linear unsteady process and the calculations were performed using the two-dimensional shallow water approximation.

In this paper we reconsider the two dimensional model of Zikanov and by use of linear stability analysis and shallow water approximation, we extract a new wave equation for interface deformation in aluminum reduction cells. This wave equation includes main parameters of cell and therefore we can see the effect of each of cell parameters on MHD instability of reduction cell. The present work is the first step in our way to the goal of developing an accurate and efficient model that includes all aspects of the cell MHD stability. The results of this model can be used as a tool for designing new reduction cells, retrofitting present cells and as a device for active control of aluminum reduction cells.

MODEL DESCRIPTION

Assumptions

In the model illustrated in Fig.1b in comparison to a real cell several simplifying assumptions are made which are presented as bellow:

1. We assume that there are not open channels between anodes in the cell and therefore we consider the cell as a closed box.
2. We ignore the compressibility of molten aluminum and cryolite and carbon-dioxide bubbles released in chemical reaction between the oxygen produced in the reduction process and carbon material of anode.

On the other hand the theoretical model is based on the following properties of flows:

A) The electrical conductivities of the aluminum, cryolite and carbon are so that

$$\sigma_{alu\,\min um} \rangle\rangle \sigma_{carbon} \rangle\rangle \sigma_{cryolite} \tag{1}$$

B) The cell is a shallow system. The typical horizontal dimensions are about $10m \times 3m$ while the mean depths of the aluminum and cryolite are about 20 cm and 5 cm. If we define h_1 and h_2 as the thicknesses of aluminum and cryolite and the typical deviation η of the interface from its equilibrium position, these values are much smaller than the typical horizontal scale of the interfacial wave λ and to the horizontal dimensions of the cell L_x, L_y, so we can use the aspect ratio as a small parameter

$$\delta \equiv \frac{(h_1, h_2, \eta)}{(L_x, L_y, \lambda)} \langle\langle 1 \tag{2}$$

Shallow Water Approximation

The assumption (2) allows transformation of the equations governing the flow into two dimensional forms:

$$\rho_a(\frac{\partial}{\partial t}U^a) + \rho_a(U^a.\nabla U^a) = -\nabla P - \rho_a g \nabla \eta + F^a - \rho_a \kappa_a U^a \tag{3}$$

$$\rho_c(\frac{\partial}{\partial t}U^c) + \rho_c(U^c.\nabla U^c) = -\nabla P - \rho_c g \nabla \eta + F^c - \rho_c \kappa_c U^c \tag{4}$$

$$\frac{\partial}{\partial t}\eta = -\nabla \cdot [(h_1 + \eta)U^a] \tag{5}$$

$$\nabla \cdot [(h_1 + \eta)U^a] + \nabla \cdot [(h_2 - \eta)U^c] = 0 \tag{6}$$

The concluded two dimensional equations are based on the shallow water approximation and all the variables are independent of the vertical coordinate z. The superscripts "a" and "c" denote the physical properties and variables are belonging to the aluminum and cryolite layers respectively. In these equations, $U^a(x,y,t)$, $U^c(x,y,t)$ are the horizontal velocities, $P(x,y,t)$ is the pressure at the interface, F^a, F^c are Lorenz forces in each layer and finally $\eta(x,y,t)$ is the interface deviation from the unperturbed flat shape. Eqs. (3), (4) are the momentum equations; the evolution equation for the interface is given by Eq. (5) and Eq. (6) expresses mass conservation. Impermeability boundary conditions for the velocity in each layer is the

$$(U \cdot \vec{n})_{sidewall} = 0 \tag{7}$$

$\vec{n}$ Is the normal unit vector at side walls.

Lorenz forces

The Lorentz forces are zero when the aluminum-cryolite interface is flat, i.e., $\eta = 0$. As soon as the interface deviates from its equilibrium state, the electric current density and magnetic field are redistributed in molten layers therefore in this condition the electrical current density and magnetic field are converted to

$$J = J_0 + j = -J_0 e_z - \sigma \nabla \phi \tag{8}$$
$$B = B_0 + b$$

Where j, b and ϕ are perturbation of electric current, magnetic field and electrical potential respectively. In this condition the perturbed Lorenz force is:

$$F = j \times B_0 + J_0 \times b + j \times b \tag{9}$$

These forces are non-local function of interface deformation and these self–amplified interactions cause the interface instability. Consider Fig.1b and let us define $\Delta\Phi$ as the electric potential drop across the electrolyte. The vertical current density perpendicular to the interface is then defined in this fluid as

$$J_\perp = -\sigma_c \nabla_\perp \phi = \sigma_c \frac{\Delta\phi}{h_2 - \eta} \approx \sigma_c \frac{\Delta\phi}{h_2}(1 + \frac{\eta}{h_2}) = J_{0\perp} + j_\perp \tag{10}$$

The contribution of the interface perturbation in the normal current density is therefore:

$$j_\perp = \frac{\sigma_c \Delta\phi}{h_2^{\,2}} \eta \tag{11}$$

While the constant vertical current without deformation of the interface is just

$$J_{0\perp} = \sigma_c \frac{\Delta\phi}{h_2} \tag{12}$$

The conservation law for electric charges may be written as

$$\nabla \cdot J = \nabla \cdot j = \nabla_\perp \cdot j_\perp + \nabla_{||} \cdot j_{||} = 0 \tag{13}$$

And may be integrated over the depth of molten aluminum layer as

$$\int_{cathode}^{hnterface} \nabla \cdot j dz = \nabla \cdot \int_{cathode}^{\text{int}erface} j_{||} dz + \int_{cathide}^{\text{int}erface} \frac{\partial j_\perp}{\partial z} dz =$$

$$\nabla_{||} \cdot j_{||} \times \int_{cathode}^{\text{int}erface} dz + \left[j_\perp \right]_{cathode}^{\text{int}erface} = \tag{14}$$

$$(h_1 + \eta)\nabla_{||} \cdot j_{||} + j_\perp^{\text{int}erface} = 0$$

Where

$$j_{||} = -\sigma_a \nabla_{||} \phi \tag{15}$$

Is the depth averaged current density in the molten aluminum and can be solved from the poisson equation:

$$\nabla^2 \phi(x, y) = -\frac{\sigma_c}{\sigma_a} \cdot \frac{\Delta\phi}{h_2^{\,2} h_1} \eta(x, y) \tag{16}$$

This equation can be written as:

$$\nabla^2 \phi = -\frac{J_0 \eta(x, y)}{\sigma_a h_2 h_1} \tag{17}$$

Since the walls of cell have very poor conductivity, the electric current doses not penetrate in cell walls and therefore the boundary condition at walls is:

$$\frac{\partial \phi}{\partial n} = 0 \tag{18}$$

In the work done by Davidson and Lindsay (1998) it was clearly shown that the most effective part of electromagnetic Lorenz force on aluminum reduction cell instability is the interaction between vertical component of unperturbed magnetic field (B_0) and horizontal component of electric current perturbation in aluminum layer i.e. $(F^a = j_{||}^a \times B_{0z} e_z)$ and F^c is negligible. Horizontal component of electric current perturbation in aluminum layer is:

$$j_{||}^a = -\sigma_a \nabla \phi \tag{19}$$

Where σ_a is the electrical conductivity of aluminum layer. Therefore Lorenz force in aluminum layer is:

$$F^a = j_{||}^a \times B_0 e_z = -\sigma_a B_{0z} (\frac{\partial \phi}{\partial y} e_x - \frac{\partial \phi}{\partial x} e_y) \tag{20}$$

$$F^c = 0$$

Interfacial wave equation

In this paper we use linear stability analysis according to this method we apply a very small perturbation on steady state of the cell at which the aluminum-cryolite interface is flat. Therefore the velocity in each layer is the sum of a constant time independent term and a time dependent perturbation term:

$$U^a = U_0^a + u^a$$
$$U^c = U_0^c + u^c \tag{21}$$

Substitution of velocity in (3) gives:

$$\rho_a (\frac{\partial}{\partial t}(U_0^a + u^a)) + \rho_a [(U_0^a + u^a).\nabla(U_0^a + u^a) = \tag{22}$$
$$-\nabla P - \rho_a g \nabla \eta + F^a - \rho_a \kappa (U_0^a + u^a)$$

By eliminating of second order terms in equations and with this assumption that the initial velocity in molten layers to be zero we have:

$$\rho_a (\frac{\partial u^a}{\partial t}) = -\nabla P - \rho_a g \nabla \eta + F^a - \rho_a \kappa u^a \tag{23}$$

If we apply this procedure for cryolite layer, the same equation results and equations (5), (6) are also converted to:

$$\rho_c (\frac{\partial}{\partial t} u^c) = -\nabla P - \rho_c g \nabla \eta + F^c - \rho_c \kappa u^c \tag{24}$$

$$\frac{\partial \eta}{\partial t} = -\nabla \cdot (h_1 u^a) \tag{25}$$

$$\nabla \cdot (h_1 u^a) + \nabla \cdot (h_2 u^c) = 0 \tag{26}$$

Eqs. (23), (24) can be written in the following form

$$\frac{\rho_c}{h_2}[\frac{\partial}{\partial t}(h_2u^c)]=-\nabla P-\rho_c g\nabla\eta+F^c-\frac{\rho_c}{h_2}\kappa(h_2u^c) \quad (27)$$

$$\frac{\rho_a}{h_1}[\frac{\partial}{\partial t}(h_1u^a)]=-\nabla P-\rho_a g\nabla\eta+F^a-\frac{\rho_a}{h_1}\kappa(h_1u^a) \quad (28)$$

By substituting the pressure term of Eq. (28) in Eq. (27) we eliminate pressure of a bow equation and we have:

$$\frac{\rho_a}{h_1}[\frac{\partial}{\partial t}(h_1u^a)]-\frac{\rho_c}{h_2}[\frac{\partial}{\partial t}(h_2u^c)]= \rho_c g\nabla\eta-F^c+\frac{\rho_c}{h_2}\kappa(h_2u^c)-\rho_a g\nabla\eta+F^a-\frac{\rho_a}{h_1}\kappa(h_1u^a) \quad (29)$$

The governing wave equation for interfacial instability in aluminum reduction cells results from applying the divergence operator to each side of above equation and by using the Eqs (25), (26).

$$\frac{\partial^2\eta}{\partial t^2}+\kappa(\frac{\partial\eta}{\partial t})-\frac{\Delta\rho g}{\alpha}\nabla^2\eta=\frac{1}{\alpha}\cdot\nabla\cdot(F^a-F^c) \quad (30)$$

In this equation $\Delta\rho=\rho_a-\rho_c$ and $\alpha=(\rho_a/h_1+\rho_c/h_2)$ condition is concluded for this wave equation comes by eliminating the pressure term of Eqs and the permeability boundary condition that was mentioned before. So we get boundary condition of this wave equation as:

$$\{\Delta\rho g\nabla\eta-(F^a-F^c)\}\cdot\vec{n}=0 \quad (31)$$

If electrical potential and magnetic field are defined as

$$B_{0z}(x,y)=B_0\cdot b(x,y)$$
$$\Phi=\frac{\sigma_a B_0}{\Delta\rho g} \quad (32)$$

Then the formulation of MHD instability in aluminum reduction cells is converted to a series of differential equations which are presented in (33).

In these equations existence of linear friction coefficient, κ, is a new term that has stabilizing effect on stability of aluminum reduction cell and in fact acts as a damping term in the wave equation. Regarded as the term of shear force in shallow water approximation diminishes, the linear friction term can satisfy the stabilizing effect of shear force.

In the derived system of equations, the function for magnetic field is obtained from the measurements of magnetic field around aluminium reduction cells that was done by moreau et al at 2002.

$$\frac{\partial^2\eta}{\partial t^2}+\kappa\frac{\partial\eta}{\partial t}-\frac{\Delta\rho g}{\alpha}\left(\frac{\partial^2\eta}{\partial x^2}+\frac{\partial^2\eta}{\partial y^2}\right)= \frac{\Delta\rho g}{(\frac{\rho_a}{h_1}+\frac{\rho_c}{h_2})}\cdot\left[\frac{\partial\Phi}{\partial x}\cdot\frac{\partial b(x,y)}{\partial y}-\frac{\partial\Phi}{\partial y}\cdot\frac{\partial b(x,y)}{\partial x}\right]$$
$$\left(\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}\right)=-\frac{J_0B_0\eta(x,y)}{\Delta\rho g h_2 h_1}$$
$$at \quad x=0,L_x \quad (33)$$
$$\frac{\partial\Phi}{\partial x}=0 \qquad \frac{\partial\eta}{\partial x}=-b(x,y)\cdot\frac{\partial\phi}{\partial y}$$
$$at \quad y=0,L_y$$
$$\frac{\partial\Phi}{\partial y}=0 \qquad \frac{\partial\eta}{\partial y}=b(x,y)\cdot\frac{\partial\phi}{\partial x}$$

RESULTS

To test the suitability of the model, we validated our results with the results of previous work that was done by zikanov et al (2000). In particular, the following parameters were used.

$L_x=11.08m$, $L_y=2.69m$, $h_1=0.18m$, $h_2=0.045m$

$\rho_a=2250kg/m^3$, $\rho_c=2075kg/m^3$

Free parameter in our formulation is the linear friction coefficient κ. In every numerical experiment, we started with a random perturbation of the interface and continued the calculation until the solution converged to a steady state (in a stable case) or until the maximum positive interface deviation exceeded the thickness of the cryolite layer (in the case of instability).

Stable Solution

In the range of stated parameters for a cell we found that there is a critical value for κ, this value is 0.0714 s^{-1}. The flow becomes unstable bellow this critical value. The oscillation start to grow in amplitude and continue in this way until the short circuit happens, when the interface touches the anode surface.

The obvious effect of the friction coefficient illustrated in figures 2, 3 that the higher friction means the more stable cell.

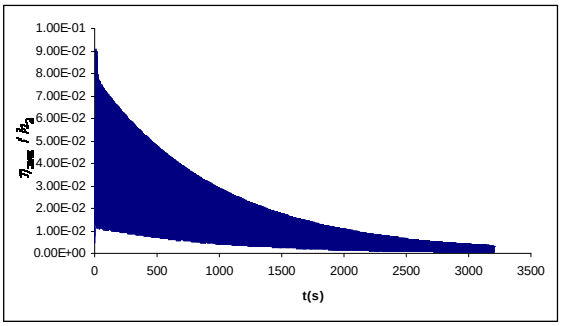

Figure 2: Stable solution calculated with $\kappa=0.0714$

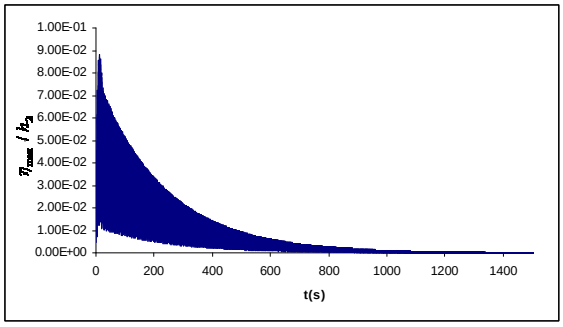

Figure 3: Stable solution calculated with $\kappa = 0.0716$

Unstable Solution

In this section, we consider the unstable interfacial wave known as the metal-pad rolling or metal-pad instability. And the effects of changing the value of different parameters of cell are determined.

If the friction is strong, the oscillations are fully suppressed during the first several periods, which lead to the steady solution discussed in previous section. When the value of κ is decreased the rate of suppression falls and the oscillations are seen for a much longer time. When the values are assumed very close to this limit, we were unable to classify the flow as stable or unstable, because there was no evidence of any grow or decay in the run time interval.

When κ is bellow the threshold value, the cell becomes unstable. The oscillations start to grow in amplitude and continue this way until the short circuit happens.

As a typical example, the unstable solution calculated with $\kappa = 0.071$ is shown in Figure 4.

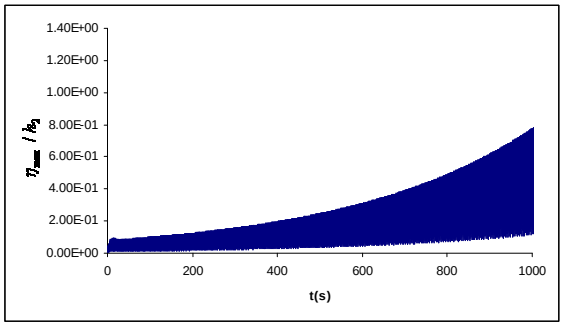

Figure 4: Unstable solution calculated with $\kappa = 0.071$

Effect of thickness of Cryolite layer on stability of cell.

One of the main parameters of a cell is the thickness of cryolite layer. As a result of this work we found that decrease in this parameter will cause the transition from stable to unstable interfacial oscillations. This point may be observed in figure 5.

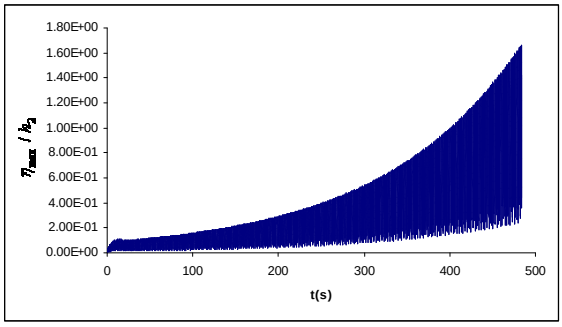

Figure 5: Unstable solution calculated with $h_2 = 0.04$

Our result on decreasing the thickness of cryolite layer is in good agreement with all previous works done in this field.

If the thickness of cryolite layer is increased, the interfacial oscillations will be suppressed more strongly. When a cell becomes unstable in operational plants, by this method the cell is returned to its stable state according to described procedure. Figure 6 shows the effect of increase of thickness of cryolite layer.

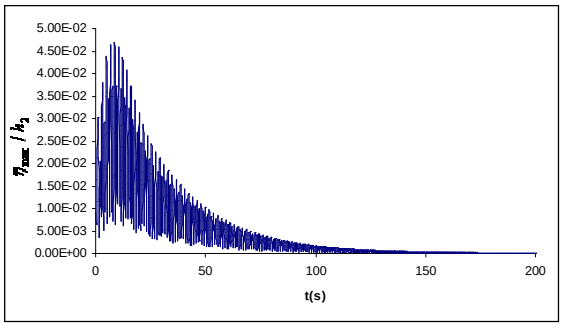

Figure 6: Stable solution calculated with $h_2 = 0.05$

High voltage decrease in cryolite layer is regarded as the main reason of high energy intensity of aluminium reduction cells therefore it would be good practice to decrease the thickness of cryolite layer as much as possible in designing the new cells. Hence a better understanding of the effect of cryolite layer seems to be necessary.

Effect of thickness of molten aluminium layer on stability of cell.

It should be noted that any change that may result from decreasing or increasing of the thickness of aluminium layer is depends on the thickness of cryolite. As soon as we decrease the thickness of aluminium layer the thickness of cryolite layer will be increased by the same amount because there is more cell volume for cryolite than the amount that is considered in modelling. This point is easily seen from figure1.b.Therefore decreasing the thickness of aluminium layer will make the cell more stable. It is shown in Figure 7.

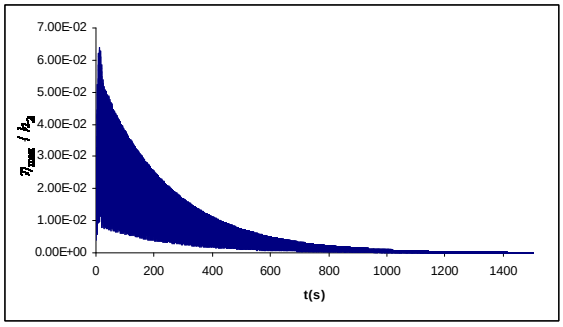

Figure 7: Stable solution calculated with $h_2 = 0.055, \quad h_2 = 0.17$

Any increase in the thickness of aluminium layer will cause a more unstable cell because it would be replaced by the same amount of thickness of cryolite. Figure 8 shows this observation.

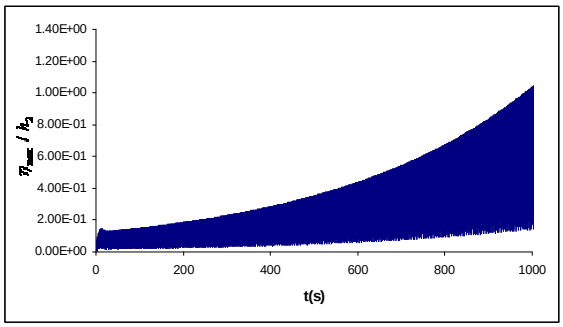

Figure8: Unstable solution calculated with $h_2 = 0.035, \quad h_2 = 0.19$

Effect of magnetic field and electric current density on the stability of cell.

Both electric current density and magnetic field have the same effect on stability of aluminium reduction cell. Decreasing these parameters will lead to increase in the stability of cell and their increase would cause the cell to be more unstable. These parameters are often constant in a cell and their values depend on the cell design and any change in their value is rarely possible. Therefore for retrofitting of existing cells we should consider other parameters of a cell.

CONCLUSION

The most important conclusion is that the model has been able to predict the instability of the interface. The results of calculation are in good agreement with results reported in previous works and theoretical statements while our model is simpler than the previous models.

Investigation of the impact of different parameters of cells on the instability phenomenon is in good agreement with nonlinear attempts in this field. The simplicity of this model helps us to have a better understanding of the phenomenon that may facilitate the design of new cells and control systems to reduce energy consumption of aluminium cell and protect it from unpleasant short circuit phenomenon that can destroy the cell.

Our model like other previous works has one drawback. It does not include the hydrodynamic effect of the channels between anodes, which may affect the flow pattern and interface stability properties. Finding the minimum possible thickness for cryolite layer for reducing energy intensity of cell, based on this model will be the next step in our study.

REFERENCES

Sele, T., (1977), "Instabilities of the metal surface in electrolytic alumina reduction cells", Metal.Trans. B, 8B, 613-618.

Urata, N., (1985), "Magnetics and metal pad instability", Light Metal, TMS, Warradale, PA, 581-589

Bojarevics, V. and M. V. Romerio., (1994), "Long wave instability of liquid metal-electrolyte interface in aluminium electrolysis cells a generation of Seles criterion", Eur. J. Mech. B, 13, 33-56.

Segatz, M. and Droste, C., (1994), "Analysis of magnetohydrodynamic instabilities in aluminium reduction cells", Light Metals, 313, 313-322.

Sneyd, A.D. and Wang, A., (1994), "Interfacial instabilities due to MHD mode coupling in aluminium reduction cells", J. Fluid Mech, 263, 342-359.

Lukyanov, a. and EL, G. and Molokov, S., (2001), "Instability of MHD-modified interfacial gravity waves revisited", Physics Letters A, 290, 165-172.

Zikanov, O. and Thess, A. and Davidson, P.A. and Ziegler, D., (2003), "A new approach to numerical simulation of melt flows and interface instability in Hall-Heroult cells", Metallurgical and Materials Transactions, 31B, 1541-1550.