Seventh International Conference on CFD in the Minerals and Process Industries
CSIRO, Melbourne, Australia
9-11 December 2009

INFLUENCE OF DRAG FORCES ON A SWARM OF BUBBLES IN ISOTHERMAL BUBBLY FLOW CONDITIONS

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ABSTRACT

Driven by the extensive demands of simulating highly concentrated gas bubbly flows in many engineering fields, numerical studies have been performed to investigate the neighbouring effect of a swarm of bubbles on the interfacial drag forces. In this study, a novel drag coefficient correlation (Simonnet et al., 2007) in terms of local void fraction coupled with the population balance model based on average bubble number density (ABND) has been implemented and compared with Ishii-Zuber densely distributed fluid particles drag model. The predicted local radial distributions of three primitive variables: gas void fraction, Sauter mean bubble diameter, and gas velocity, are validated against the experimental data of Hibiki et al. (2001). In general, satisfactory agreements between predicted and measured results are achieved by both drag force models. With additional consideration for closely packed bubbles, the latest coefficient model by Simonnet et al. (2007) shows considerably better performance in capturing the reduction of drag forces incurred by neighbouring bubbles.

NOMENCLATURE

a_{if}	Interfacial area concentration
C_D	Drag coefficient
$C_{D\infty}$	Drag coefficient of isolated bubbles
D	Inner diameter of the pipe
D_s	Bubble Sauter mean diameter
E_o	Eötvos number
F_i^D	Drag force
n	Average number density of gas phase (bubble)
Re_m	Mixture Reynolds number
u_∞	Velocity of isolated bubbles in a quiescent liquid
$\vec{u}$	Velocity vector

Greek Symbols

α	Void fraction
α_{gm}	Maximum packing value
α_{loc}	Local void fraction
μ	Viscosity
μ_m	Mixture viscosity
ρ	Density
σ	Surface tension
ϕ_n^{RC}	Coalescence rate due to random collision
ϕ_n^{TI}	Break-up rate due to turbulence impact

Subscripts

g	Gas
i	Index of gas/liquid phase
l	Liquid

INTRODUCTION

Two-fluid model based on the inter-penetrating continua approach is probably considered to one of the practical and accurate macroscopic formulations of gas-liquid flow systems. Herein, exchanges that occur at the interface between the two phases are explicitly accounted and the dynamics of the interaction are fully described through appropriate constitutive relations governing the inter-phase mass, momentum and energy exchanges. The existence of these terms is one of the most important characteristics of the two-fluid model formulation since they determine not only the degree of mechanical and thermal non-equilibrium but also the rate of phase changes between phases.

In the absence of heat and mass transfer, the complexity of the problem reduces to consideration of only the momentum exchange term, which is typical of isothermal bubbly flow. The interfacial transfer in the conservation momentum equation generally involves the consideration of drag and non-drag interfacial force densities. The interfacial drag force is a result of the shear and form drag of the fluid flow, which depends on the drag coefficient as well as the interfacial area concentration. For solid particles, the drag coefficient depends only on the characteristics of the flow surrounding the particle and is primarily a function of the particle Reynolds number and of the turbulence intensity of the continuous phase (Bertola et al., 2004). For bubbles, the behaviours are however complicated due to three important aspects. Firstly, when the liquid is pure enough, it has the possibility to slip along the surface of the bubbles. This is in contrast to flow past rigid (solid) bodies where the no-slip condition is imposed. Secondly, almost all the inertia is contained in the liquid for bubbly flow due to the relative weak density of bubbles compared to that of the liquid; thus inertia induced hydrodynamic forces is particularly important in the prediction of bubble motion. Thirdly, bubbles have a tendency to deform due to coalescence and break-up; the changes of bubble shape add new degrees of freedom to an already complex problem (Magnaudet and Eames, 2000).

The mutual interaction among bubbles can also have a significant influence on the drag force (Behzadi et al., 2004; Bertola et al., 2004; and Simonnet et al., 2007). Figure 1 illustrates the profile of the liquid velocity passing through the case of multi-bubbles in comparison with one of a single bubble with uniform incoming flow. It demonstrates that the profile of the liquid velocity distribution along radial position can be significantly affected by adding swarm of bubbles. With higher void fraction, bubbles replace and narrow more space of cross section for liquid, which increase main liquid velocity due to liquid mass conservation. However, liquid velocity close to pipe wall and bubble surface is comparatively slow because of the viscous force. Thus, great liquid velocity gradient or even small eddies possibly be produced in some local position. On the other hand, with high void fraction, bubbles have tendency to be closer and it may be significantly affected by the presence of a slanted wake behind other bubbles in a multi-bubble system, which may add complexity to numerical simulation of multi-bubble systems. Finally, these entire non-uniform and complicated liquid behaviours in turn dramatically affect the air-water slip velocity induced drag force as well as the bubble wake or shear induced lift force acting on the two-phase flow.

Over the years, several models have been proposed to calculate the drag force for bubbles at high void fractions. Ishii and Zuber (1979) have categorized the bubbly flow behaviours into different regimes. A mixture viscosity model has been developed to obtain each drag coefficient correlations for the individual flow regimes. CFX commercial software drag coefficient model based on Ishii and Zuber (1979)'s drag formulations has been widely applied and can yield reasonable predictions for a range of flow conditions. Many recent investigations by Rusche and Issa (2000), Behzadi et al. (2004) and Simonnet et al. (2007) have nevertheless focused on the modelling of suitable drag coefficient multiplier across a wide range of void fractions and different flow regimes. Through this approach, the ratio of the drag coefficient to its single dispersed element value can be fitted to a function of the phase fraction, i.e. $C_D/C_{D\infty}=f(\alpha)$ where $C_{D\infty}$ is the drag coefficient of an isolated bubble in an infinite medium. Simonnet et al. (2007) have developed a novel drag coefficient expression by correlating the drag coefficient multiplier with exhaustive experimental data. The improved expression to predict two-phase flow from bubbly to slug transitional regime is investigated in the present study. Within this regime, bubbles are generally found to be highly distorted and closely packed in contrast to the bubbly flow regime of which the bubbles are normally spherical in shape and allowed to move freely (Hibiki et al., 2001 and Cheung et al., 2007).

The primary aim of this paper is to compare the relative merits and capabilities applying two drag coefficient formulations commonly used Ishii and Zuber (1979) model and recently proposed by Simonnet et al. (2007) to evaluate the drag force in the conservation momentum equation. In order to predict the dynamic changes of the interfacial structure, the use of first-order equation to characterise the transport of interfacial area concentration (Hibiki and Ishii, 2002, Yao and Morel, 2004) or averaged bubble number density (ABND) (Cheung et al., 2007) has sufficed. The ABND model (Cheung et al. 2007) is thus employed to predict the local bubble distribution in an upward flow channel and special emphasis is directed

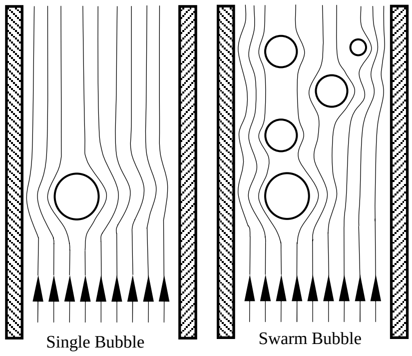

Figure 1: Profile of liquid velocity passing through single bubble and multi-bubbles

towards investigations on bubbly-to-slug transitional regime (cap-bubbly flow condition) where high void fraction and high liquid velocity exist. Numerical predictions through these two drag coefficient correlations are compared against experimental data of isothermal gas-liquid bubbly flow in a vertical pipe performed by Hibiki et al. (2001).

MATHMATICAL MODELS

Two-Fluid and ABND models

The two-fluid model conservation equations for mass and momentum for bubbly flows can be written as:

$$\frac{\partial(\rho_i\alpha_i)}{\partial t}+\nabla\cdot(\rho_i\alpha_i\vec{u}_i)=0 \tag{1}$$

$$\frac{\partial(\rho_i\alpha_i\vec{u}_i)}{\partial t}+\nabla\cdot(\rho_i\alpha_i\vec{u}_i\vec{u}_i)=-\alpha_i\nabla P+\alpha_i\rho_i\vec{g}$$
$$+\nabla\cdot\left[\alpha_i\mu_i^e\left(\nabla\vec{u}_i+(\nabla\vec{u}_i)^T\right)\right]+F_i^D+F_i^{ND} \tag{2}$$

where F_i^D is the interfacial drag force density while F_i^{ND} consists of the non-drag force contributions.

For dispersed isothermal bubbly flow, assuming a single bubble size given by the bubble Sauter mean diameter, the average bubble number density n for bubbly flow can be defined as:

$$n=\frac{\alpha_g}{\pi D_S^3/6} \tag{3}$$

And the average bubble number density transport equation takes the form:

$$\frac{\partial n}{\partial t}+\nabla\cdot(\vec{u}_g n)=\phi_n^{RC}+\phi_n^{TI} \tag{4}$$

where $\vec{u}_g$ is the mean gas velocity. The phenomenological mechanisms of coalescence and breakage are affected through the source and sink terms ϕ_n^{RC} and ϕ_n^{TI} of which they are due to random collision, turbulent induced breakage. The Yao and Morel (2004) model is adopted in the present study. More details regarding the model can be referred in Cheung et al. (2007).

Interfacial Momentum Transfer due to Drag

The inter-phase momentum transfer between gas and liquid due to the drag force resulted from shear and from drag and can be modelled in terms of the interfacial area concentration a_{if} and slip velocity $(\vec{u}_g - \vec{u}_l)$ as:

$$F_{l\to g}^{D} = \frac{1}{8} C_D a_{if} \rho_l \left| \vec{u}_g - \vec{u}_l \right| \left(\vec{u}_g - \vec{u}_l \right) \tag{5}$$

It should be noted that $F_{l\to g}$ in the above equation depicts the momentum transfer from the gas phase to the liquid phase. In the present study, two correlations of several distinct Reynolds number regions for individual bubbles proposed by Simonnet el al. (2007) and Ishii and Zuber (1979) correlation are employed to evaluate the drag coefficient C_D in equation (5).

Ishii and Zuber (1979) drag coefficient correlation takes into account for multi-bubble effects by considering different bubble shape regimes; such as: dense spherical particle regime, dense distorted particle regime and dense spherical cap regime.

Dense Spherical Particle Regime

$$C_D(sphere) = \frac{24}{Re_m}\left(1 + 0.15\, Re_m^{0.687}\right) \tag{6}$$

Dense Distorted Particle Regime

$$C_D(ellipse) = \frac{2}{3}\sqrt{Eo}\, E \tag{7}$$

Dense Spherical Cap Regime

$$C_D(cap) = \frac{8}{3} E' \tag{8}$$

where Re_m is mixture Reynolds number. More information can be found in Ishii and Zuber (1979)

In equation (7), the dense distorted particle regime drag coefficient model takes the form of a multiplying factor E, which is given in terms of the void fraction as

$$E = \left[\frac{1 + 17.67 f(\alpha_g)^{6/7}}{18.67 f(\alpha_g)} \right]^2 \tag{9}$$

Where

$$f(\alpha_g) = \frac{\mu_l}{\mu_m}(1 - \alpha_g)^{1/2} \tag{10}$$

And, Eo represents the Eotvos number which is defined by

$$Eo = \frac{g\left(\rho_l - \rho_g\right) D_s^2}{\sigma} \tag{11}$$

where σ is the surface tension coefficient.

For dense spherical cap regime, the multiplication factor E' takes however the form:

$$E' = (1 - \alpha_g)^2 \tag{12}$$

As implemented within ANSYS CFX 11 (ANSYS, 2005. CFX-11 User Manual) the regime selection is based on

$$C_D = \begin{cases} C_D(sphere) & if\, C_D(sphere) \geq C_D(ellipse) \\ \min\left(C_D(ellipse), C_D(cap)\right) & if\, C_D(sphere) \leq C_D(ellipse) \end{cases} \tag{13}$$

On the other hand, through recent experimental investigation by Simonnet et al. (2007), a novel drag correlation for pure air-water systems has been proposed. It can be written as

$$C_D = C_{D\infty} E'' \tag{14}$$

where $C_{D\infty}$ is the drag coefficient of an isolated bubble in an infinite medium, which can be obtained through the balance of buoyancy, drag and gravitational as

$$C_{D\infty} = \frac{4}{3} \frac{\rho_l - \rho_g}{\rho_l} g D_s \frac{1}{u_\infty^2} \tag{15}$$

In the above equation, u_∞ represents the velocity of an isolated bubble in a quiescent liquid which can be calculated using the correlation of Jamialahmadi et al. (1994):

$$u_\infty = \frac{u_{b1} u_{b2}}{\sqrt{u_{b1}^2 + u_{b2}^2}} \tag{16}$$

where

$$u_{b1} = \frac{1}{18} \frac{\rho_l - \rho_g}{\mu_l} g D_s^2 \frac{3\mu_g + 3\mu_l}{3\mu_g + 2\mu_l} \tag{17}$$

$$u_{b2} = \sqrt{\frac{2\sigma}{D_s(\rho_l - \rho_g)} + \frac{g D_s}{2}} \tag{18}$$

In equation (14), the multiplication factor E'' according to Simonnet et al. (2007) is

$$E'' = (1 - \alpha_g)\left[(1 - \alpha_g)^m + \left(4.8 \frac{\alpha_g}{1 - \alpha_g}\right)^m\right]^{-2/m} \tag{19}$$

Where, m is set a value of 25. The above modification is valid for a wide range of void fractions and across different flow regimes.

Interfacial Momentum Transfer due to Non-Drag Forces

Alongside with the drag force, other interfacial non-drag forces considered in the present study conclude lift force, wall lubrication force and turbulent dispersion force. More details can be found in Cheung et al. (2007).

EXPERIMENTAL AND NUMERICAL DETAILS

The numerical model has been validated against experiments conducted by Hibiki et al. (2001). The test section was a round tube made of acrylic with an inner diameter D=50.8mm and length of 3061mm. Local flow measurements using the double sensor and hot film anemometer probes were performed at three axial (height) locations of z/D=6.0, 30.3 and 53.5 and 15 radial locations of r/R=0 to 0.95. Solutions to the two sets of balance equations for mass and momentum of each phase are sought. Radial symmetry has been assumed in which numerical simulations were performed on a 60 degree radial sector of the pipe with symmetry boundary conditions at both vertical sides. A three-dimensional mesh containing 108,000 hexagonal elements is generated over the entire pipe domain. At the inlet of the test section, uniformly distributed superficial liquid and gas velocities, void fraction are specified in accordance with the flow condition described. As the diameter of the injected bubbles is unknown, inlet bubble size of 3mm is adopted based on experimental information at location of z/D=6.0. The initial bubble number density was calculated using Equation (3). Details of the boundary conditions have been summarised in Table 1. The Shear Stress Transport (SST) turbulent model is employed in the present study. As depicted in Fig. 2, four flow conditions employed for validation are in the bubbly-to-slug transitional regime, particularly cap bubbles are observed in flow condition of $\langle j_f \rangle$ =0.986 m/s and $\langle j_g \rangle$ =0.321 m/s. As demonstrated in Ho and Yeoh (2005), coalescence of capped bubbles and its interaction with bubbly flow mixtures may become significant in bubbly-to-slug transitional flow regime causing noticeable discrepancy between predicted results and measured data.

Table 1: Bubbly flow conditions and its inlet boundary conditions employed in the present study.

Superficial liquid velocity $\langle j_f \rangle$ **(m/s)**	**Superficial gas velocity** $\langle j_g \rangle$ **(m/s)**	
Hibiki et al. (2001) experiment		
0.491	0.129	0.190
[$\alpha_g\|_{z/D=0.0}$ (%)]	[20.0]	[25.0]
[$D_S\|_{z/D=0.0}$ (mm)]	[3.0]	[3.0]
0.986	0.242	0.321[b]
[$\alpha_g\|_{z/D=0.0}$ (%)]	[20.0]	[25.0]
[$D_S\|_{z/D=0.0}$ (mm)]	[3.0]	[3.0]

[b] Cap bubbles were experimental observed in this flow condition

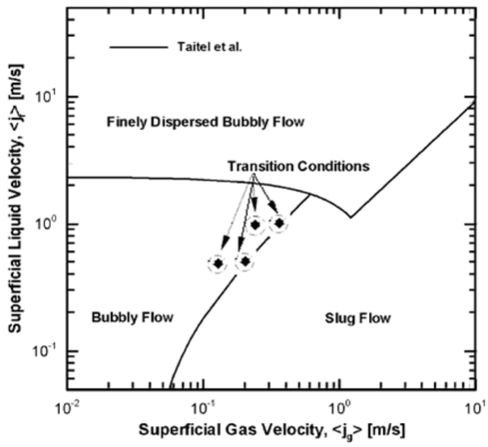

Figure 2: Map of tube flow regime and transition flow conditions studied in the present study

RESULT AND DISCUSSION

Simulations of the isothermal upward bubbly flow in a tube have been carried out for four different operations, intended to cover a range of gas and liquid velocity. By comparison of simulation results with experimental data, values of the various adjustable parameters in the model are determined, so as to give reasonable results over all operating conditions. In the present study, the breakage and coalescence calibration factors are set as 0.6 and 0.1 respectively to balance whole systems' bubble number density. The similar calibration factors adjustments were employed in research of Chen et al. (2005) and Olmos et al. (2001).

Void fraction distribution

Figure 3 shows the void fraction distribution obtained from Simonnent et al. (2007) and Ishii and Zuber (1979) drag coefficient correlations comparing with the experimental data at the dimensionless axial position Z/D = 53.5. In the present research, the first three flow conditions (Figure.3a, b, c) characterised as "wall peaking" phase distribution are recorded and have been captured very well by both drag coefficient models.

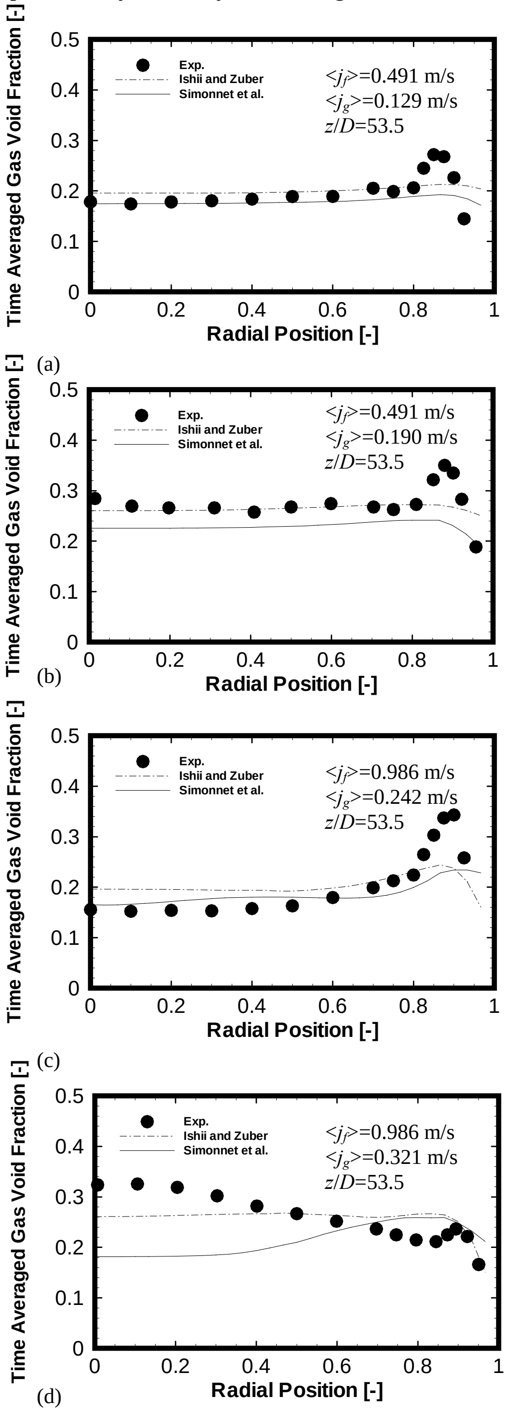

Figure 3: Predicted radial void fraction distribution and experiment data of Hibiki et al. (2001).

However, the two drag coefficient correlations both show the under-predicted void fractions at the core (Fig.3d) for flow condition of $\langle j_f \rangle$ =0.986 m/s and $\langle j_g \rangle$ =0.321 m/s, in which "transition" phase distribution and cap bubble have been observed. Since lateral lift force presents the migration for bubbles toward the pipe centre or wall, it becomes the dominating factor to govern void fraction distributions. In this research, Tomiyama (1998)'s lift coefficient correlation based on consideration of bubble deformation has been employed. However, this coefficient was developed only from experimental data of single bubble rising through an infinite stagnant liquid in a vertical pipe. The latest lift force correlation proposed by

Hibiki and Ishii (2007) in multi bubble system has also been tested, however no significant improvement has been observed. According to the research by Hibiki and Ishii (2007), the lift force in multi-bubble system is affected by many factors, such as relative velocity between two phases, shear rate of liquid, bubble rotational speed, surface boundary condition, void fraction. In this study, Ishii and Zuber (1979) drag coefficient correlation seems to give better prediction in void fraction. It may contribute from other factors rather than relative velocity.

Sauter mean bubble diameter

Figure 4 illustrates the predicted and measured Sauter mean bubble diameter distributions. Except for the flow condition of $\langle j_f \rangle$ =0.986 m/s and $\langle j_g \rangle$ =0.321 m/s, the Sauter mean bubble diameter appeared almost uniform along the radial direction. Owing to the tendency of small bubbles migrating towards the wall, highly concentred bubbles near the wall have greater possibility to colloid forming slightly larger bubbles. For cases of flow condition of $\langle j_f \rangle$ =0.986 m/s and $\langle j_g \rangle$ =0.242 m/s (Figure 4c), both Simonnet et al. (2007) and Ishii and Zuber (1979) drag coefficient give reasonably good prediction compared with experimental data. For other two "wall peak" flow cases (Figure 4a, b), slightly over-prediction are given by both models, however Simonnet et al. (2007) drag correlation generally give better numerical prediction compared with Ishii and Zuber (1979) drag coefficient. For studied flow condition of $\langle j_f \rangle$ =0.986 m/s and $\langle j_g \rangle$ =0.321 m/s, cap-bubble with wide range of bubble size along radial position were observed in experiment (Figure 4d). It may be difficult to catch by ABND model since this population balance model only presents behaviour of average bubble diameter and normally gives flat bubble size distribution prediction. Furthermore, ignoring source term of wake entrainment by Yao and Morel (2004) model used in this study possibly is another reason for discrepancy between numerical prediction and experimental observation since cap bubble has stronger wake entrainment to suck the following small bubbles.

Time-averaged gas velocity

The local radial gas velocity profiles from the two drag coefficient correlations and the experimental data at the measuring station are shown in Figure 5. With additional consideration of the neighbouring bubbles, the Simonnet et al. (2007) drag coefficient correlation based on the introduction of local void fraction has the tendency to give comparatively better prediction of the gas velocity profiles in all four flow conditions, compared to the Ishii and Zuber (1979) correlation. However, for the study case of $\langle j_f \rangle$ =0.986 m/s and $\langle j_g \rangle$ =0.321 m/s, the Simonnent et al. (2007) drag coefficient correlation still under predicted local radial gas velocity at the centre of pipe. Generally in vertical bubbly flow, the density difference buoyant force is regarded as the driving force for bubbly dispersed phase rising. For "core peak" flow condition of $\langle j_f \rangle$ =0.986 m/s and $\langle j_g \rangle$ =0.321 m/s, high void fraction concentrate was observed in the centre of pipe which may lead stronger driving force (density difference buoyant force) per unit volume for bubble dispersed phase. On the other hands, highly concentrated void fraction has tendency that bubbles are protected under their neighbour's wake area and possibly have weaker drag influence. Furthermore, the complex bubble coalescence/breakage, sophisticated bubble deform and strong turbulent flow mixing may be difficultly to be numerically simulated in a constitutive equation theoretically at this stage.

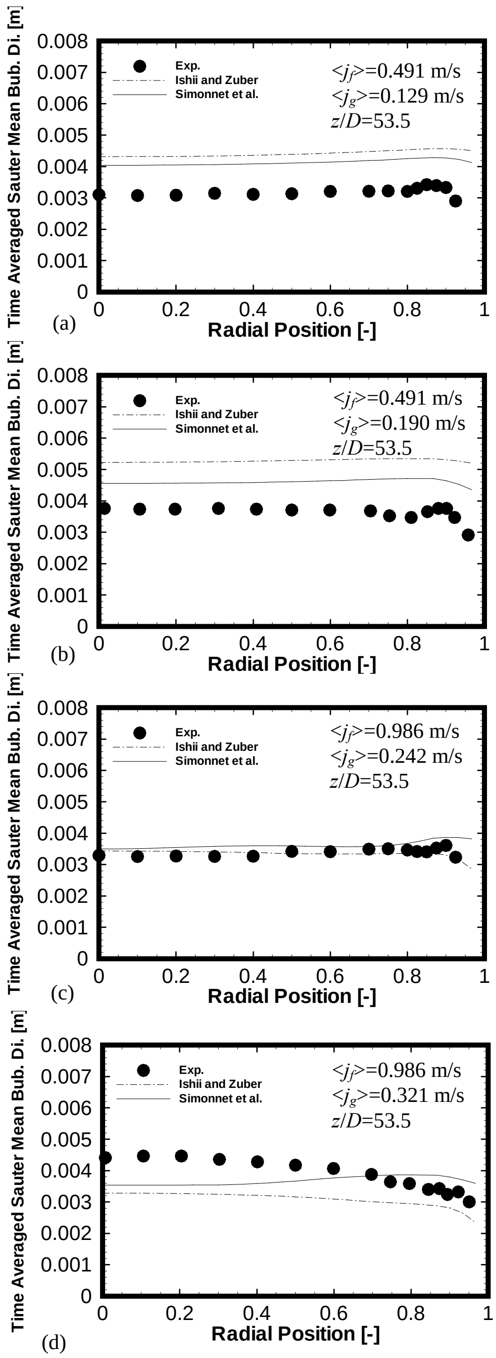

Figure 4: Predicted Sauter mean bubble diameter distribution and experiment data of Hibiki et al. (2001).

CONCLUSIONS

An averaged one-group population balance approach, the average bubble number density (ABND) transport equation, coupled with the Eulerian-Eulerian two-fluid model is presented in this paper to handle the gas-liquid bubbly flows under isothermal conditions, particularly under bubbly-to-slug transition regime. Based on ABND model, drag mechanisms respectively developed by Simonnet et al. (2007) and Ishii and Zuber (1979) are compared against experimental data conducted by Hibiki et al. (2001) under various flow conditions. Local radial distributions of three primitive variables: void fraction, Sauter mean bubble diameter and gas velocity, are compared. In general, both of the drag coefficient

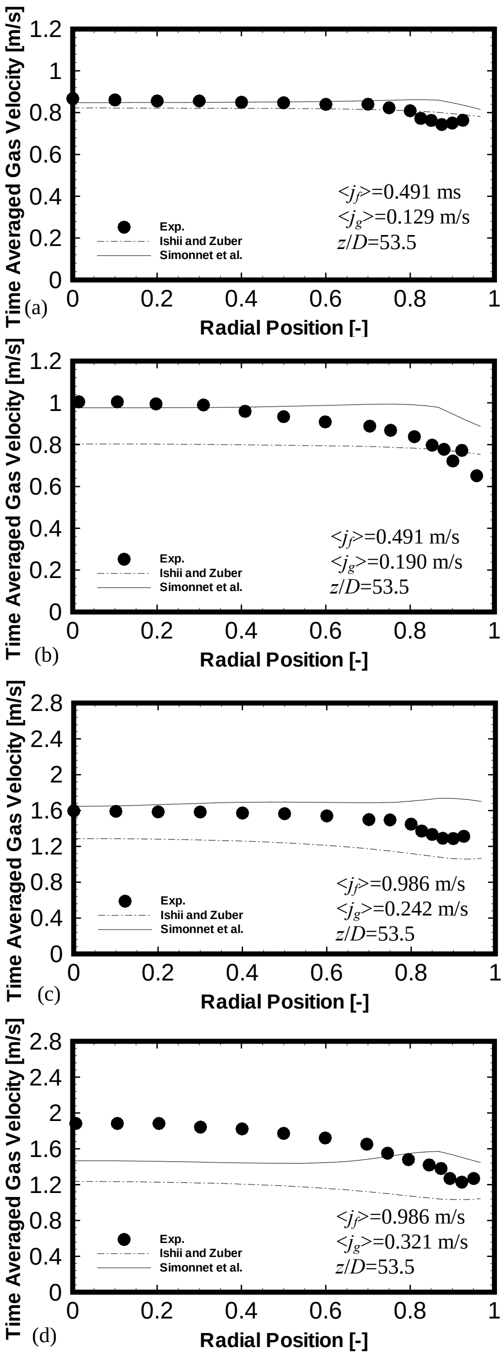

Figure 5: Predicted radial gas velocity profile and experiment data of Hibiki et al. (2001).

correlations predictions yield fair agreement with experimental results. Due to drag force compared with other interfacial force has closer relationship with liquid/gas velocity; Simonnet et al. (2007) model which accounts additional considerations of neighbour bubbles presents relatively better predictions in gas velocity. Such finding ascertains that the effect of neighbouring bubbles could be influential in high void fraction flow condition where bubbles are closely packed. Although additional considerations were attempted to incorporate into the drag coefficient, some notable discrepancies between the numerical and experimental results still remain suggesting more in-depth investigation on the drag coefficient of a swarm of bubbles should be carried out in future.

ACKNOWLEDGMENTS

The financial support is provided by the Australian Research Council (ARC-Discovery DP0877734) is gratefully acknowledged and the first author is also supported by RMIT University through International Post Graduate Research Scholarship (IPRS).

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