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MODELLING THE MULTIPHASE FLOW IN DENSE MEDIUM CYCLONES

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ABSTRACT

Dense medium cyclone (DMC) units are widely used to upgrade run-of-mine coal in the coal preparation industry. Their simple design belies the complex flow patterns that existing within each unit. These are developed due to the size and density distributions of the feed, presence of medium solids, and the turbulent vortex that is formed. The modelling of flow in a DMC is very challenging. Recently, the so-called combined computational fluid dynamics (CFD) and discrete element method (DEM) (CFD-DEM) was extended from two-phase flow to model the complex multiphase flow in DMCs at UNSW. In the CFD-DEM model, the flow of coal particles is modelled by DEM and that of medium flow by CFD, allowing consideration of medium-coal mutual interaction and particle-particle collisions. In the DEM model, Newton's laws of motion are applied to individual particles, and in the CFD model the local-averaged Navier-Stokes equations combined with the Volume of Fluid (VOF) and Mixture multiphase flow models are solved. The application to the DMC studies requires extraordinarily intensive computational effort, particularly for industrial scale DMCs. Therefore, various simplified versions have been proposed, corresponding to the approaches such as Lagrangian Particle Tracking method where dilute phase flow is assumed so that the interaction between particles can be ignored, one-way coupling where the effect of particle flow on fluid flow is ignored, and the use of concept of parcel particles whose properties are empirically determined. In this paper, the features and applicability of these approaches are discussed.

INTRODUCTION

The dense medium cyclone (DMC) is a high-tonnage device that has been widely used to upgrade run-of-mine coal in the coal industry by separating gangue from product coal. The flow in a DMC is very complicated with the presence of swirling turbulence, an air core and segregation of medium and coal particles. It involves multiple phases: air, water, coal and magnetite particles of different sizes, densities and other properties. The flow in DMCs is so complex that the numerical modelling of DMCs is very challenging.

Recently, the so-called combined computational fluid dynamics (CFD) and discrete element method (DEM) (CFD-DEM) approach has been extended from two-phase flow to model the complex multiphase flow in DMCs at UNSW (Chu et al., 2009a). The application of CFD-DEM method to the DMC studies requires extraordinarily intensive computational effort, particularly for industrial scale DMCs. Therefore, various simplified versions have been proposed, corresponding to the approaches such as CFD and Lagrangian Particle Tracking (CFD-LPT) method where dilute phase flow is assumed so that the interaction between particles can be ignored (Suasnabar and Fletcher, 2003; Narasimha et al., 2007; Wang et al., 2009a; Wang et al., 2009b), one-way coupling where the effect of particle flow on fluid flow is ignored (Chu et al., 2009b), and the use of concept of parcel particles whose properties are empirically determined (Chu et al., 2009c). All of these approaches have been used at UNSW. In this paper, the work done at UNSW on the simulation of DMCs is summarised and the features and applicability of the approaches are discussed.

MODEL DESCRIPTION

In the CFD-DEM model, the motion of particles is modelled as a discrete phase, by applying Newton's laws of motion to individual particles, while the flow of fluid is treated as a continuous phase, described by the local averaged Navier-Stokes equations on a computational cell scale. The approach has been recognised as an effective method to study the fundamentals of particle-fluid flow by various investigators (Yu and Xu, 2003; Zhu et al., 2007) and its mathematical formulation has been well documented (Tsuji et al., 1992; Xu and Yu, 1997; Xu et al., 2000; Zhu et al., 2007). Chu and Yu (2008) extended this model to study complex particle-fluid flow. However, only a single fluid phase is present in those studies. In this work, the continuous phase represents a mixture of water, air, and magnetite particles of different sizes and densities.

The continuous medium flow is calculated from the continuity and the Navier-Stokes equations based on the local mean variables over a computational cell, which are given by

$$\frac{\partial(\rho_f \varepsilon)}{\partial t} + \nabla \cdot (\rho_f \varepsilon \mathbf{u}) = 0 \qquad (1)$$

and

$$\frac{\partial(\rho_f \varepsilon \mathbf{u})}{\partial t} + \nabla \cdot (\rho_f \varepsilon \mathbf{u}\mathbf{u}) = -\nabla P - \mathbf{F}_{p-f} + \nabla \cdot (\varepsilon \boldsymbol{\tau}) + \rho_f \varepsilon \mathbf{g} \qquad (2)$$

where ε, $\boldsymbol{u}$, t, ρ_f, p, $\mathbf{F}_{f-p}$, $\boldsymbol{\tau}$, and $\boldsymbol{g}$ are, respectively, porosity (equal to volume fraction of fluid), fluid velocity and time, fluid density and pressure, volumetric fluid-particle interaction force, fluid viscous stress tensor, and acceleration due to gravity. $\mathbf{F}_{f-p} = \sum_{i=1}^{k_c} \mathbf{f}_{p-f,i}$, where $\mathbf{f}_{p-f,i}$ is the total fluid force on particle i and k_c is the number of particles in a CFD cell. The flow solved in Eqs. (1) and (2) represents the mixture flow of medium and air, and was obtained by use of the Volume of Fluid (VOF) and Mixture Multiphase Flow (MMF) models in a commercial CFD software package, i.e. Fluent (see Steps 1 and 2 in Figure 1). The details of the medium flow calculation and its validation can be found elsewhere (Wang et al., 2007a; Chu et al., 2009b).

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On the basis of the fluid flow obtained above, as the third step of the whole modelling approach shown in Figure 1, the flow of coal particle can be modelled either by the LPT or DEM (Cundall and Strack, 1979). LPT model can be treated as an extreme case of DEM when only one particle is tracked in the DMC and the impact of solid/solid and solid/fluid interactions are ignored. According to DEM, a particle in a DMC was considered to have two types of motion: translational and rotational. During its movement, the particle may collide with its neighbouring particles or with the wall and also interact with the surrounding fluid, through which momentum and energy are exchanged. At any time t, the equations governing the translational and rotational motions of particle *i* in this two-phase flow system are:

$$m_i \frac{d\mathbf{v}_i}{dt} = \mathbf{f}_{p-f,i} + m_i\mathbf{g} + \sum_{j=1}^{k_i}\left(\mathbf{f}_{c,ij} + \mathbf{f}_{d,ij}\right) \qquad (3)$$

and

$$I_i \frac{d\boldsymbol{\omega}_i}{dt} = \sum_{j=1}^{k_i}\left(\mathbf{T}_{c,ij} + \mathbf{T}_{r,ij}\right) \qquad (4)$$

where m_i, I_i, $\mathbf{v}_i$, and $\boldsymbol{\omega}_i$ are, respectively, the mass, moment of inertia, translational and rotational velocities of particle *i*. The forces involved are the gravitational force, $m_i\mathbf{g}$, inter-particle forces between particles i and j which include the contact forces $\mathbf{f}_{c,ij}$, and viscous damping forces $\mathbf{f}_{d,ij}$, and the total particle-fluid interaction forces, $\mathbf{f}_{p-f,i}$, which is the sum of various particle-fluid forces, including viscous drag force and pressure gradient force in the current case. Torques, $\mathbf{T}_{c,ij}$, are generated by the tangential forces and cause particle i to rotate, because the inter-particle forces act at the contact point between particles i and j and not at the particle centre. $\mathbf{T}_{r,ij}$ are the rolling friction torques that oppose rotation of the ith particle. Trial simulations indicated that other particle-fluid forces, such as virtual mass force and lift force, could be ignored in this work. The fluid properties used to calculate the particle-fluid interaction forces are those relating to the individual phases in the mixture, i.e., water, air and magnetite particles of different sizes. The details of the calculation of the forces in Eqs.(1)-(4) can be found in our previous studies (Zhou et al., 1999; Xu et al., 2000).

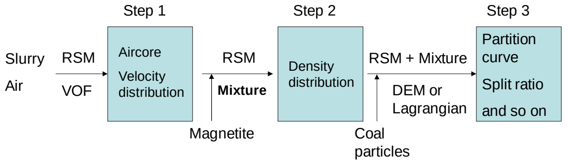
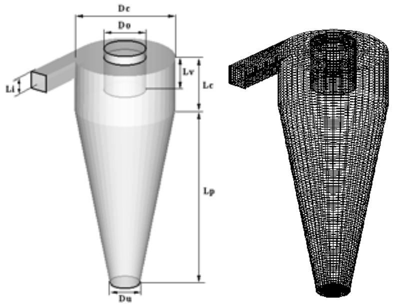


Figure 1: Schematic diagram of the modelling approach.

The modelling of the solid flow by DEM is at the individual particle level, whilst the fluid flow by CFD is at the computational cell level. Their two-way coupling (the fluid forces on particles and the reaction of particles on fluid) can be achieved as follows. At each time step, based on fluid flow field, DEM will give information, such as the positions and velocities of individual particles, to allow the evaluation of porosity and the volumetric particle-fluid interaction force in a computational cell. CFD will then use these data to update the fluid flow field which then yields new particle-fluid interaction forces acting on individual particles. Incorporation of the resulting forces into DEM will produce information about the motion of individual particles for the next time step. According to this coupling method, the reaction of particles on medium flow can be considered.

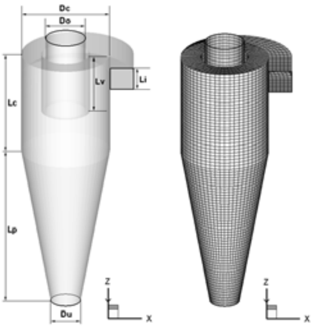
SIMULATION CONDITIONS

In this work, the results of four previous studies were summarised. In the four studies, CFD-LPT, CFD-DEM one-way coupling model, CFD-DEM two-way coupling model with parcel particles concept and CFD-DEM two-way coupling model are used respectively. For the purpose of model validation of the medium flow, the DMC used in Subramanian's experiments (Subramanian, 2002) was used in the studies by use of CFD-LPT and CFD-DEM one-way coupling methods. For the purpose of model validation of coal flow, the industrial data for a large diameter DMC (Rong, 2007) is used in the studies by use of CFD-DEM two-way coupling method with parcel particles concept and CFD-DEM two-way coupling method. The geometries and mesh representations of the two DMCs are shown in Figures 2 and 3 respectively. The other details of the simulation conditions can be found elsewhere in (Chu et al., 2009a; Chu et al., 2009b; Chu et al., 2009c; Wang et al., 2009a; Wang et al., 2009b). In the simulation, the performance of the DMC is evaluated in terms of partition curve, cut relative density (D_{50}) and Ecart probable (Ep). D_{50} is defined as the relative density of particles that have equal probability of reporting to either underflow or overflow. Ep = (D_{75}-D_{25})/2, where D_{75} and D_{25} are the relative densities for which 75% and 25% of feed particles report to underflow respectively. They are the parameters commonly used to determine the separating performance of a DMC (Wood, 1990).



(a) (b)

Figure 2: Geometry (a) and mesh representation (b) of the simulated DMC in the studies by use of CFD-LPT and CFD-DEM one-way coupling methods (Dc=350 mm).



(a) (b)

Figure 3: Geometry (a) and mesh representation (b) of the simulated DMC in the studies by use of CFD-DEM two-way coupling with parcel particles concept and CFD-DEM two-way coupling methods (Dc=1000 mm).

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RESULTS

CFD-LPT Model

Figure 4 shows the comparison of the measured (Subramanian, 2002) and predicted density distributions using the CFD method. The profiles are very similar. More comparison can be found in (Wang et al., 2007a). Based on the results of CFD modelling, the trajectory of a coal particle is tracked by LPT model. As shown in Figure 5, heavy particles (RD = 1.6 and 1.7) are rejected through the spigot while light particles (RD = 1.2, 1.3 and 1.4) escape from the vortex finder. An approximate cut point particle (RD = 1.5) is initially dragged down by the external downward flow and then drawn up by the inner upward flow.

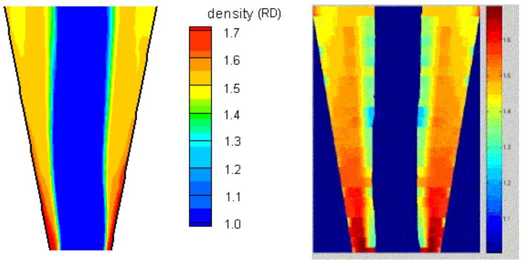


Figure 4: Comparison of the predicted (left) and measured (right) medium density distribution.

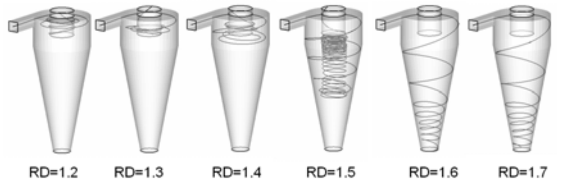
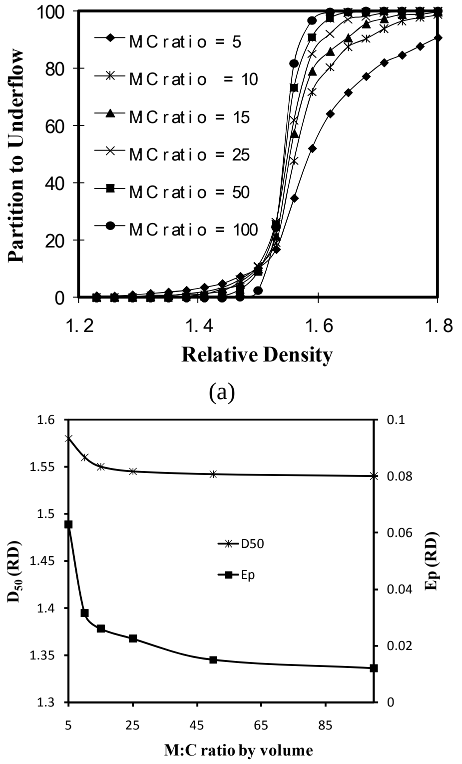


Figure 5: Typical trajectories of coal particles with different relative densities when the medium feed relative density is 1.467.

CFD-DEM one-way coupling model

Medium-to-coal ratio by mass (M:C ratio) has been known as one of the key factors in DMC operation (Killmeyer, 1982). Its effect on the performance was examined by the present CFD-DEM one-way coupling method. Note that this effect cannot be studied by the LPT model. Figure 6 (a) shows that there is almost no tail in the partition curve when the M:C ratio (100:1) is high. The tail increases as the M:C ratio decreases. Figure 6 (a) also shows that there are some heavy particles reporting to overflow when the M:C ratio is 5, which may be caused by the relatively small spigot (=0.3 Dc) of the DMC. Heavy particles may overload at the spigot when its size is small. Another reason is probably due to the collision of the light and heavy particles; the random collision near the vortex finder may cause some heavy particles to flow to the centre where a large drag force can encourage them to flow upward. As shown in Figure 6 (b), when the M:C ratio is higher than 15, D_{50} and Ep increases slightly as the M:C ratio decreases. However, as the M:C ratio further decreases from 15 to 5, D_{50} and Ep increase significantly. This effect is qualitatively comparable to those observed in the experiments or in practice (Wood, 1990). Under the current simulation conditions, the M:C ratio affects partition performance because its decrease increases the intensity of inter-particle interaction. The increased inter-particle contacts come from the “blocked flow” of light particles toward the center or heavy particles toward the wall of the DMC. They may simply increase the possibility of the random movements of particles and hence deteriorate the partition performance.



(a)

(b)

Figure 6: Simulated partition curve (a), separation density D_{50} and Ep (b) in the DMC under different M:C ratio when particle size is 5 mm.

Surging has been reported as an important phenomenon in DMC operation and its control is important since it results in a large portion of coal product reporting to rejects (Wood, 1990). Surging was considered to be caused by the instability of medium flow under certain conditions, e.g. when the DMC was not properly designed (Wang et al., 2007b; Wang et al., 2009a). However, by the same token, it is also related to the instability of particle flow. This may happen when particle-particle interaction is extremely strong, although it is often not considered in the traditional modelling. The present simulations indicate that the unstable solid flow can be found if the M:C ratio is too low by use of CFD-DEM one-way coupling method.

Figures 7 and 8 show the surging behaviour when the M:C ratio is 3 and particle size is 8 mm. It can be seen that solid particles gradually accumulated in the DMC, although a limited amount of solids does exit through the vortex finder or spigot. However, once the accumulation reaches a critical level, particles will discharge through the spigot all of a sudden. This periodic accumulating-discharging pattern is identified as one of the surging phenomena in DMC operation. To explore this phenomenon further, Figure 7 shows a cycle of this pattern, corresponding to the period of 2-6 s in Figure 8. It can be seen from the two figures that at the initial stage (t<2.0 s), the solids in the DMC increase almost linearly, which means there are very few particles flowing out.

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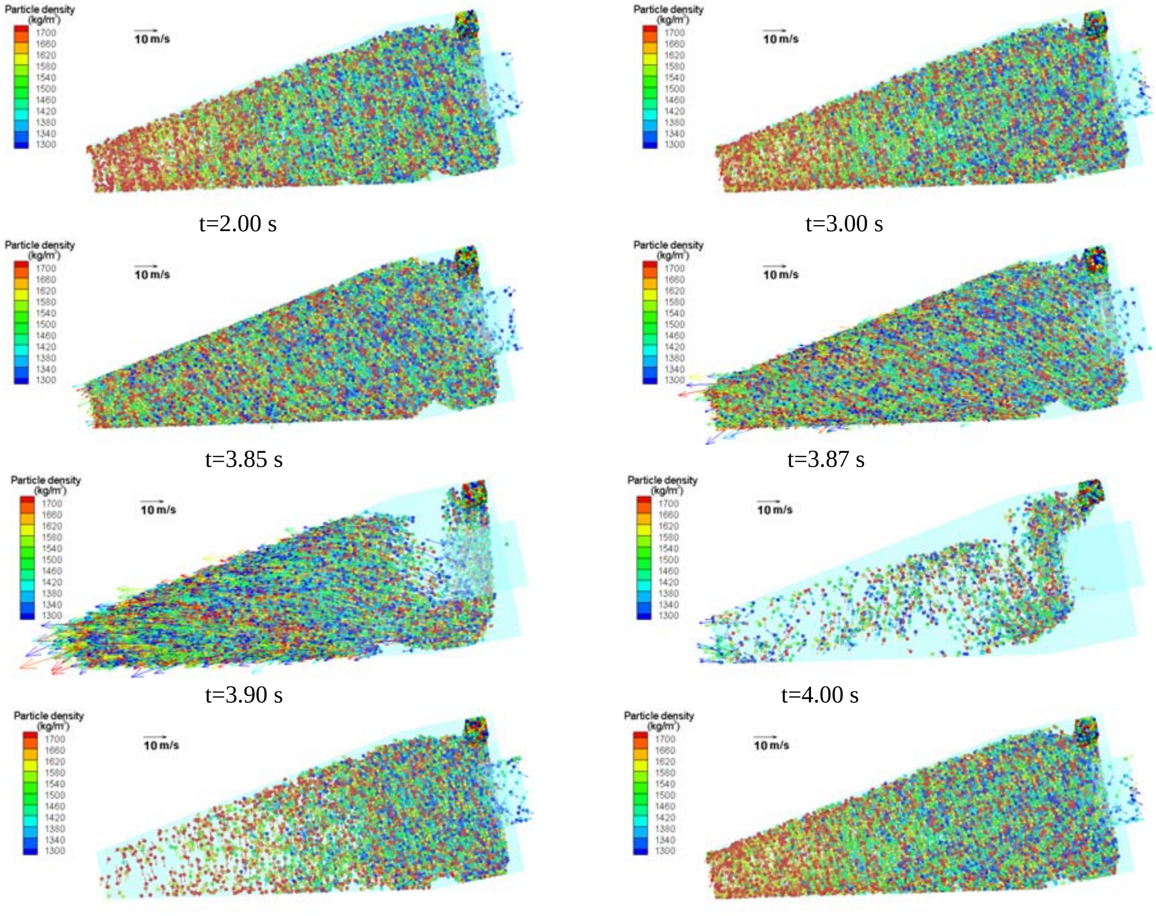

t=2.00 s t=3.00 s

t=3.85 s t=3.87 s

t=3.90 s t=4.00 s

t=5.00 s t=6.00 s

Figure 7: Snapshots showing the surging phenomenon of solid flow in the DMC when particle size is 8 mm and M:C ratio is 3 (colour represents different densities and the vector is the velocity of coal particle).

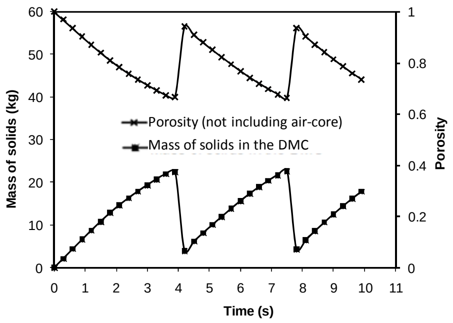

Figure 8: Variations of total mass of solid and porosity with time in the DMC corresponding to the surging case shown in Figure 7.

At t=3.85 s, the axial velocity of particles at the spigot increases obviously. This suggests that there are more particles flowing out through the spigot. Correspondingly, the accumulation rate decreases, as shown in Figure 8. At the same time, however, many light particles are misplaced into the spigot and there are almost no particles reporting to the overflow. At t = 3.9 s, when the porosity in the DMC excluding air core area is as low as 0.65, which may correspond to the onset of dilatancy (Onoda and Liniger, 1990) or global jamming (Dong et al., 2009), the DMC simply empties itself in order to generate space to accommodate the incoming particles for another surging cycle.

CFD-DEM two-way coupling model with parcel particles concept

In order to simulate an industrial-scale DMC, the concept of parcel particles, similar to that used by Patankar and Joseph (2001), was adopted (Chu et al., 2009c). According to this concept, a group of real particles with some common properties (size and density in the current case) can be represented by one parcel-particle. At a given point in the fluid flow, the acceleration of the parcel-particle due to fluid forces is assumed to be the same as that of the group of real particles it represents. However, the acceleration due to inter-particle forces and particle-wall forces are calculated according to the properties of the parcel-particle which, however, need to be determined a priori. In this study, for simplicity, the material properties of a parcel-particle were assumed to be the same as those of real particles.

Figure 9 shows that the simulated partition curves are compared with experimental measurements quantitatively well. It can be seen that the simulated partition curves for particles with sizes from 4 to 11 mm agrees well with experimental data. For smaller particles, as shown in Figures 8 (b) and (c), the simulated partition curves are both to the left of those measured.

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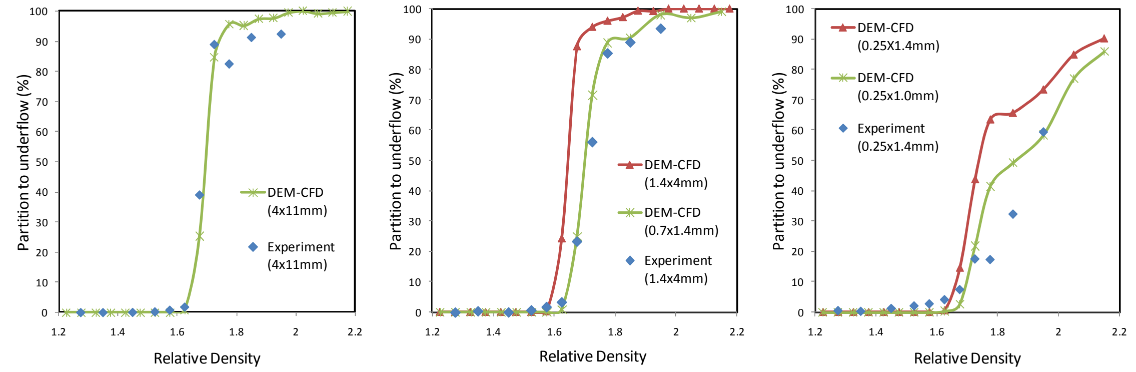

(a) (b) (c)

Figure 9: Comparison of the simulated and measured partition curves of particles with different sizes in the DMC when the M:C ratio is 5.4: (a), 4-11mm; (b), 1.4-4 mm; and (c), 0.25-1.4 mm.

After validation, the CFD-DEM two-way coupling method with parcel particles concept is used to investigate the effect of coal particle density distribution which is an important parameter representing the difference between two major coal-types, i.e. coking coal and thermal coal. Three density distributions (a, b, and c) are considered, with "a" has the least and "c" has the most near-gravity density particles among the three density distributions.

Figure 10 shows the general flow patterns of particles for the three density distribution considered. They are consistent with the earlier identified phenomenon that low density coal particles mainly accumulate in the upper part of the DMC and exit from overflow through vortex finder while high density particles mainly move downwards to the underflow along the cyclone wall. The figure also shows that the particle flow patterns with different particle density distributions are different from each other. There are more near-gravity particles in the conic region when there are more near-gravity particles in the feed. This is consistent with the phenomenon that near-gravity particles have a longer residence time in the cyclone (Wood, 1990; Chu et al., 2009b).

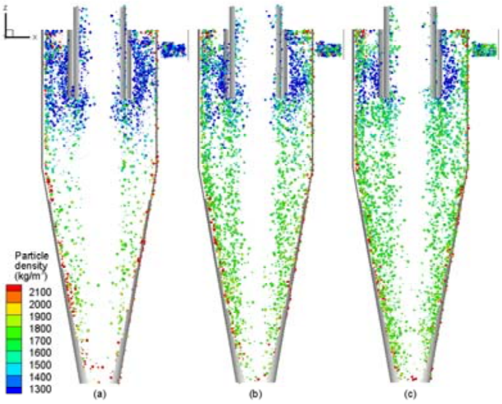

Figure 10: The flow pattern of coal particles (0.25-25 mm) at a central vertical slice (of thickness 10% Dc) in the DMC with different coal particle density distributions when the M:C ratio is 5 (at t = 30.0 s).

CFD-DEM two-way coupling model

It is found that both the medium and solids flow change significantly when coal particle density distribution is changed while the mass flowrates of both medium and solid phases are kept constant (Chu et al., 2009c). This actually suggests that coal particles of different density have different impacts on the medium-coal flow of DMCs. In this section, in order to confirm this finding and understand the effect of particle density on the coal-medium flow in a DMC, a CFD-DEM two-way coupling study (Chu et al., 2009a) was carried out under controlled conditions to examine the flow in a typical DMC by feeding only particles of one specific density into the DMC in each run. The M:C ratio is kept constant as 19 in each run.

The flow of medium is important since it largely controls the flow of coal particles (Chu et al., 2009c). The operational head is a common macroscopic parameter to describe medium flow and defined as the pressure drop between the inlet and outlet of the vortex finder of the DMC divided by medium feed density, gravity acceleration and DMC diameter. As shown in Figure 11, the impact of particle loading on the head is highly sensitive to particle density even though the mass flowrates of particles are the same for all of the runs. The operational head decreases as particle density increases (other conditions are fixed). It can be seen that the head is 5.98 before coal particles are added into the DMC, i.e., under pure medium flow conditions. After adding coal, the head is either higher or lower than 5.98 depending on the density of the particles added. Compared with the head under pure medium flow conditions, the head increases by 22% while the particle RD is 1.2 and decreases by 5% while the particle RD is 2.2. Generally speaking, the addition of coal particles of high densities (>1.9 in this case) reduces the head while that of low densities (<1.9 in this case) increases the head.

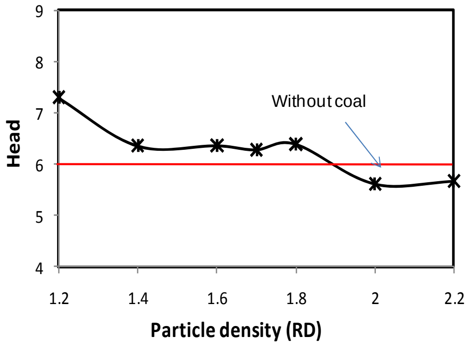

Figure 11: Simulated operational head as a function of particle RD.

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CONCLUSION

It is shown that CFD-LPT, CFD-DEM one-way coupling, CFD-DEM two-way coupling with parcel particles concept and CFD-DEM two-way coupling models can all be used in the simulation of the medium-coal flow in DMCs. Their features and capabilities are summarised as follows:

- CFD-LPT model ignores particle-particle interaction and the reaction of particles on fluid. It uses the least computational effort but is theoretically only suitable for dilute flow. Thus it can be used to obtain medium flow and track single particle flow in DMCs. It can also be used to investigate the effects of parameters other than M:C ratio and coal type qualitatively.
- CFD-DEM one-way coupling model considers particle-particle interaction on the base of CFD-LPT model but ignores the reaction of particles on fluid. It can be used to study the effect of particle-particle interaction in DMCs.
- CFD-DEM two-way coupling model with parcel particles concept considers both parcel-fluid and parcel-parcel interactions but some of the properties of the parcel particles need to be determined empirically. It can simulate large-scale DMC systems but it may not be suitable generally.
- CFD-DEM two-way coupling model has the least model assumptions and has been successfully used to simulate the medium-coal multiphase flow in a DMC but it requires the most computational effort.

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