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A CONTROL VOLUME FINITE ELEMENT SCHEME FOR ANALYSIS OF HEAP LEACHING

Peyman MOSTAGHIMI[1,2*], Brendan S. TOLLIT[2], Stephen J. NEETHLING[1], Gerard J. GORMAN[2], and Christopher C. PAIN[2]

[1] Rio Tinto Centre for Advanced Mineral Recovery, Department of Earth Science and Engineering, Imperial College London, SW7 2AZ, UNITED KINGDOM

[2] Applied Modelling and Computation Group, Department of Earth Science and Engineering, Imperial College London, SW7 2AZ, UNITED KINGDOM
*Corresponding author, E-mail address: peyman@ic.ac.uk

ABSTRACT

Accurate determination of the fluid flow within heap leaching is critical to understanding and improving performance of the process. Numerical methods have the potential to assist by modelling the process and studying the transport phenomena within the porous medium. This paper presents a numerical scheme to solve for unsaturated incompressible flow in porous media with applications to heap leaching. The classic Richard's equation is solved for the porous medium representing a heap. The numerical method is implemented in Fluidity (AMCG, 2012), an open source and general purpose finite element/volume model. It has the capability of solving a number of different governing equations for fluid flow and accompanying field equations on arbitrary unstructured meshes. A control volume finite element scheme is employed to simulate the two phase flow of air and leaching solution in the porous media. An implicit pressure explicit saturation formulation (IMPES) is used to decouple the pressure and saturation equations. Pressure is discretized using a control volume finite element method (CV-FEM) and saturation with a node centred control volume method. The model is verified by modelling the Buckley-Leverett problem where a quasi-analytical solution is available. The method is applied to two phase flow of air and leaching solution in a simplified two dimensional heap geometry. We also demonstrate the potential to achieve high spatial accuracy at low computational cost through the use of the anisotropic mesh adaptivity.

NOMENCLATURE

d_p	diameter of particles
K	permeability
k_a	absolute permeability
k_r	relative permeability
p	pressure
p_c	capillary pressure
p_e	entry capillary pressure
p_{nw}	non-wetting pressure
p_w	wetting pressure
S	saturation
S_n	non-wetting saturation
S_r	residual saturation
S_w	wetting saturation
t	time
t_d	dimensionless time
$\underline{u}$	velocity
u_t	total velocity
α	pore size distribution index
φ	porosity
μ	viscosity
ρ	density

INTRODUCTION

Numerical modelling of multiphase flow in porous media is of great importance in many fields of engineering and sciences (Bear, 1988). Originally these models are based on the finite difference method (Aziz and Settari, 1986). However, finite difference methods typically lack the flexibility required to represent complex geometries and material properties of porous media and capture the multiscale evolving features associated with the flow in porous media. Unstructured finite volume/element methods are capable of capturing complex three dimensional geometries and can be combined with adaptive mesh methods to achieve high fidelity models. While the formulation and methods described in this paper can be generalized to different areas of transport in porous media, here we focus on the application of a control volume finite element scheme to simulate and analyse the transport of multiphase fluids in heap leaching.

Heap leaching is one of the most important methods to extract metals such as copper, zinc, and gold from ores. It consists of two main processes: multiphase fluid flow in the heap and the physio-chemical reaction between the ores and the leaching solution. These two processes can be studied separately assuming that the reaction process does not affect the flow pattern (Cariaga et al., 2005). The accurate simulation of the flow is of great importance since other aspects of the process, such as rate of reaction, depend highly on it. In addition, rapid and accurate numerical simulation of heap leaching phenomena is a cost effective method to design and optimize the operational processes and irrigation strategies. In this paper we focus on the numerical simulation of multiphase flow within a heap where the displacing phase is the leaching solution and the displaced phase is air.

The governing equations for multiphase flow in porous media consist of Darcy's law along with the conservation of mass for each phase, which together form the classic Richard's equation. Different approaches have been applied to solve for flow in a heap leaching process. Munoz et al (1997) proposed a two dimensional mathematical model based on the finite difference discretization of Darcy's law and validated their results by comparing against a radial flow experiment. McBride et

al. (2012) applied a finite volume method to solve for the flow and modelled a gold oxide heap successfully. Cariaga et al. (2005) employed a mixed hybrid finite element approach to solve for liquid and air flow in a two dimensional pilot-scale heap. McBride et al (2006) gave a review on some common numerical techniques to solve for the flow in a heap leaching process. Furthermore, Mellado et al. (2009) proposed an analytical approach based on the simple Bernoulli flow to model the recovery in heap leaching.

In this paper, a control volume finite element discretization is developed and applied to solve for the flow in porous media. The numerical model is implemented in Fluidity (AMCG, 2012). Fluidity is an open source CFD code capable of solving a number of different governing equations for fluid flow and accompanying field equations on arbitrary unstructured meshes. It contains advanced numerical features such as parallel mesh adaptivity and has been applied to many geophysical and industrial applications (Davies et al., 2011, Piggott et al., 2008, Pain et al., 2001). The primary variables are chosen to be the pressure of air and the saturation of liquid. An IMplicit Pressure Explicit Saturation (IMPES) method is used to decouple pressure and saturation. In this method first introduced by Stone and Garder Jr (1961), Sheldon et al. (1959), the flow equations are split into an equation for pressure and an equation for saturation. These two equations are discretized in time using implicit and explicit approximations respectively. The saturation equation is spatially discretized using node centred control volume formed around the unstructured finite element mesh. The face values are determined through an upwind scheme. The pressure equation is spatially discretized using a continuous control volume finite element method (CV-FEM) such as to be consistent with the discrete saturation equation. To verify the model, we compare our numerical results against the quasi-analytical solution for the Buckley-Leverett two phase flow problem (Buckley and Leverett, 1942). The advantage of this numerical approach is its consistent discretization of the pressure and saturation equations and its capability to model the heap leaching process in full scale with comprehensive characterization of air and liquid flow.

MULTIPHASE FLOW IN POROUS MEDIA

Governing Equation

The Darcy's law along with the conservation of mass equation form the basis of multiphase incompressible flow in porous media (Bear, 1988). Assuming no sources or sinks in the domain, the mass conservation for each phase can be written as

$$\frac{\partial(\phi\rho_l S_l)}{\partial t} + \nabla\cdot\left(\rho_l \underline{u}_l\right) = 0 \tag{1}$$

where φ is the porosity of the porous medium, ρ is the density, S is the saturation, and $\underline{u}$ is the volumetric velocity of phase l. Darcy's law for phase l is given by

$$\underline{u}_l = -\frac{K_l}{\mu_l}(\nabla p_l - \rho_l \underline{g}) \tag{2}$$

where p is the pressure, K is the permeability, and μ is the isotropic viscosity of phase l. In addition to the conservation of mass and Darcy's law, the following relationships must be satisfied to close the system of equations:

$$\sum_l S_l = 1 \tag{3}$$

$$p_c(S_w) = p_n - p_w \tag{4}$$

where p_c is the capillary pressure, p_n and p_w are pressure of non-wetting phase and wetting phase respectively.

Permeability

Permeability (K) is a measure of the ability of a porous medium to allow the flow transit through. It can be decomposed into absolute permeability (k_a) and relative permeability (k_r). Absolute permeability is a characteristic property of the porous medium which is not affected by the presence of another fluid. However, when more than one fluid exists in the porous medium, the ratio of the effective permeability of each phase to the absolute permeability determines the relative permeability such that

$$K_l = k_a k_{rl} \tag{5}$$

Absolute permeability like other physical properties of porous materials is a function of the complex micro structure of the medium (Dullien, 1992). It can be related to other properties of the porous medium that can be measured more conveniently such as porosity and particle size. The classical Kozeny-Carman equation is a common way of estimating absolute permeability and it is capable of predicting permeability of monosized packs of spherical particles with a reasonable accuracy (Carman, 1937). However, Garcia et al. (2009) showed that the Kozeny-Carman relation overpredicts the permeability of a porous medium consisting of irregular particles. They proposed a correlation for absolute permeability as a function of the harmonic mean diameter of the particles d_p and the porosity of the medium:

$$k_a = 0.11 d_p^2 \phi^{5.6} \tag{6}$$

In this paper, their correlation for absolute permeability is applied to model a heap. Average diameter of particles in a heap leaching process is typically 0.5cm and porosity of a heap is around 0.35. This leads to an absolute permeability of 7.7×10^{-9} m^2.

In contrast to the absolute permeability, relative permeability needs to be updated as the liquid passes through the heap. In application, the relative permeability of a particular phase is presented as a function of saturation. For the heap model, we apply a quadratic relative permeability-saturation correlation as follows:

$$k_{rl} = \left(\frac{S_l - S_{lr}}{1 - S_l}\right)^2 \tag{7}$$

where S_{lr} is the residual saturation of phase l.

Capillary Pressure

The pressure of two immiscible fluids has a jump discontinuity across their interface in a porous medium. This pressure difference is called capillary pressure and defined as $p_c=p_n-p_w$. In the heap leaching process, air is considered as the non-wetting phase and the leaching solution is the wetting phase. In application, capillary pressure can be correlated as a function of saturation.

Brooks and Corey (1964) presented a correlation for the capillary pressure such that

$$p_c(S_w) = p_e\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{-\alpha} \tag{8}$$

where p_e is the entry capillary pressure and α is the pore size distribution index. Entry capillary pressure can be calculated by dividing the surface tension of the non-wetting phase in contact with the wetting phase to the characteristic length of the porous medium. In the heap leaching application, it can be approximated as the surface tension of water in contact with air (0.07 N/m) divided by the average particles' diameter (0.5 cm). This gives us an entry capillary pressure of p_e=14 *Pa* and we choose α=0.5 for the pore size distribution index.

NUMERICAL METHOD

This section provides a brief description of the temporal and spatial discretization methods applied for modelling of the incompressible multiphase flow in porous media.

Temporal Discretization

Assuming incompressible fluids and substituting in for the u_l from equation 2 in equation 1, the phase conservation equation is obtained as:

$$\frac{\partial(\phi S_l)}{\partial t} - \nabla\cdot\left(\frac{K_l}{\mu_l}\nabla p_l - \frac{K_l\rho_l}{\mu_l}\underline{g}\right) = 0 \tag{9}$$

A global continuity equation is then formed via summing equation 9 for all phases and using the constraint from equation 3:

$$\nabla\cdot\left(\frac{K_l}{\mu_l}\nabla p_l - \frac{K_l\rho_l}{\mu_l}\underline{g}\right) = 0 \tag{10}$$

The primary variables to be solved prognostically are chosen to be p_n and S_w from which p_w and S_n can be obtained diagnostically through equations 3 and 4.

We use equation 10 to solve for the pressure using a control volume weighting function and finite element basis functions. Then to update the saturation, we solve equation 9 using a control volume scheme with the same order as the weighting functions employed for pressure. For temporal discretization, we employ an IMplicit Pressure Explicit Saturation (IMPES) method. In this method, to calculate the pressure at time step n+1, equation 10 is used while the saturation is at previous time level, n:

$$\nabla\cdot\left(\frac{K_l^n}{\mu_l}\nabla p_l^{n+1} - \frac{K_l^n\rho_l}{\mu_l}\underline{g}\right) = 0 \tag{11}$$

Solving for the pressure implicitly, provides enough information to solve for the saturation explicitly. So, we apply equation 9 to update saturation explicitly and assume that the porosity is invariant in time:

$$\frac{\phi(S_l^{n+1} - S_l^n)}{\Delta t} = \nabla\cdot\left(\frac{K_l^n}{\mu_l}\nabla p_l^{n+1} - \frac{K_l^n\rho_l}{\mu_l}\underline{g}\right) \tag{12}$$

Spatial Discretization

Since the pressure is discretized by a finite element basis and saturation is solved on node centred control volume basis, they can be written as:

$$p^n = \sum_{j=1}^{M}\hat{p}_j^n N \tag{13}$$

$$S^n = \sum_{j=1}^{M}\hat{S}_j^n M_b \tag{14}$$

where N is standard Lagrangian finite element basis, M_b is 1 inside the control volume constructed around the node j (see Figure 1) and zero everywhere else. The p_j and S_j are the pressure and saturation at degrees of freedom to be determined which for the basis functions chosen also correspond to the node j values.

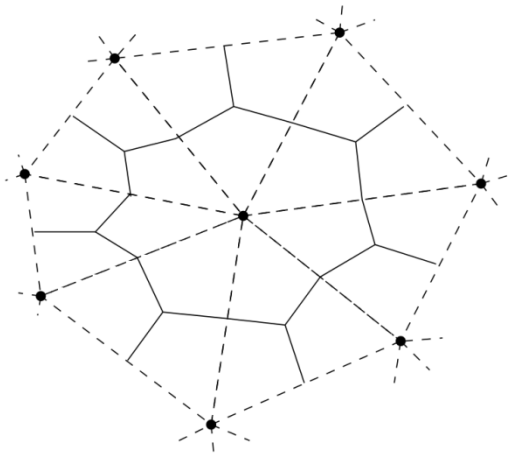


Figure 1: The node centred control volume mesh (shown by solid lines) is constructed around continuous finite element mesh (shown by dashed lines) by connecting the centroids of the neighbouring finite elements to the edge midpoints (AMCG, 2012).

To generate a linear system for pressure a Petrov-Galerkin Weighted Residual method is used. This involves multiplying equation 11 by a control volume based weight function and integrating over Ω to give:

$$-\int_{\Omega} M_b\nabla\cdot\left(\sum_l\left(\frac{K_l^n}{\mu_l}\nabla p_l^{n+1} - \frac{K_l^n\rho_l}{\mu_l}\underline{g}\right)\right)dV = 0 \tag{15}$$

Since M_b is 1 inside control volume b and zero everywhere else, we obtain:

$$-\int_{\Omega_{CV_b}}\nabla\cdot\left(\sum_l\left(\frac{K_l^n}{\mu_l}\nabla p_l^{n+1} - \frac{K_l^n\rho_l}{\mu_l}\underline{g}\right)\right)dV = 0 \tag{16}$$

where the integration is now solely over the local control volume. Now, applying the divergence theorem to relate the curvature of the field inside the control volume to the flux through its surface yields:

$$-\int_{\Gamma_{CV_b}}\underline{n}.\nabla\cdot\left(\sum_l\left(\frac{K_l^n}{\mu_l}\nabla p_l^{n+1} - \frac{K_l^n\rho_l}{\mu_l}\underline{g}\right)\right)d\Gamma = 0 \tag{17}$$

where Γ_{CV} is the control volume surface bounding CV_b and $\underline{n}$ is the unit outward pointing normal. The assembly of the linear system for the solution of the pressure is now

reduced to integrations over the control volume surfaces for each term. The gradient of the pressure basis function can be readily evaluated on the control volume surfaces as they are internal to the finite element. The absolute permeability and viscosity are taken to be element wise such that they can be easily evaluated on the control volume surfaces. For incompressible flow conditions considered here the buoyancy term is constant across the domain. The relative permeability is represented with the same control volume basis set as the saturation thus a face value is required in the assemble process. Here a simple upwind scheme is taken where an estimate of the upwind direction is deduced from a finite element interpolation from an effective velocity.

To generate a linear system for saturation a standard node centred control volume weighting and basis is used to obtain

$$\int_{\Omega_{CV_b}} \frac{\phi S^{n+1}}{\Delta t} dV = \int_{\Gamma_{CV_b}} \underline{n} \cdot \left(\frac{K_l^n}{\mu_l} \nabla p_l^{n+1} - \frac{K_l^n \rho_l}{\mu_l} \underline{g} \right) d\Gamma + \int_{\Omega_{CV_b}} \frac{\phi S^n}{\Delta t} dV \quad (18)$$

As for the pressure equation all terms associated with the advection term can be readily evaluated at the control volume surfaces. For consistency the relative permeability face values used in the pressure matrix are also used for the saturation advection matrix. This ensures that the discrete saturation equations summed over all phases produces the discrete pressure equation.

RESULTS AND DISCUSSIONS

The Buckley-Leverett problem

The Buckley-Leverett model is a one dimensional two phase flow in a homogeneous medium for which the quasi-analytical solution can be derived (Helmig, 1997). In this model, the porous medium is initially saturated with phase 1. Phase 2 is introduced to the medium by a constant flux from the left. We compare our numerical results against the quasi-analytical solution to show the numerical accuracy of the method and check its order of convergence. The one dimensional domain is discretized into equidistant elements. We assume that the viscosity of phase 1 and 2 are the same and relative permeability is obtained from following equations:

$$k_{r1} = S_1^2 \quad (19)$$

$$k_{r2} = (1 - S_1)^2 \quad (20)$$

In the Buckley-Leverett problem, a dimensionless time is defined as

$$t_d = \frac{u_t t}{\phi X} \quad (21)$$

where u_t is the total velocity of both phases and X is the length of the domain. We chose a uniform permeability of 10^{-10} m^2, a uniform porosity of 0.5 and a total velocity of u_t =1 *m/s*. The viscosity of the both phases is 10^{-4} *Pa.s*.

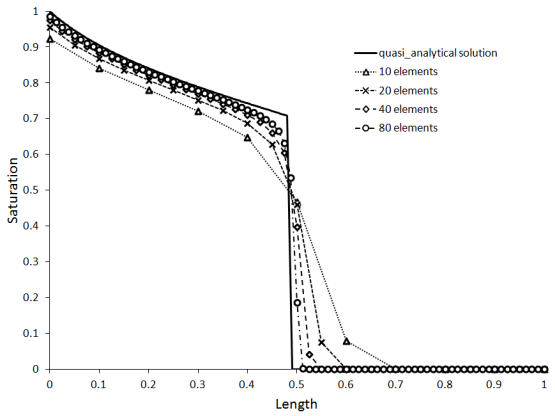


Figure 2: Saturation profile for 10, 20, 40, and 80 elements in comparison with the quasi-analytical solution.

Figures 2 shows the comparison of the saturation profiles obtained from numerical simulation using 10, 20, 40, and 80 elements with the quasi-analytical solution after dimensionless time of 0.4. The numerical simulation shows a very good agreement with the quasi-analytical solution. The simulation results improve for finer meshes the front saturation is matched very closely for 80 elements.

The heap model

We simulate the two phase flow of air and leaching solution in a heap assuming that the transport occurs in a vertical two dimensional plane. Therefore, we model the flow on a two dimensional geometry as shown in Figure 3.

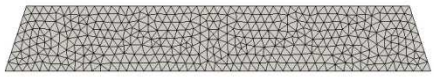


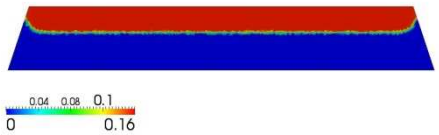
Figure 3: Two dimensional heap geometry used for flow simulation.

The width and height of the heap are assumed to be 100 and 15 meters respectively. The porosity and permeability are assumed to be uniform in space and invariant in time - φ=0.35 and k_a=7.7×10^{-9} m^2. Density and viscosity of air are 1.2 $kg.m^{-3}$ and 1.8×10^{-5} *Pa.s* respectively. For the leaching solution, these two parameters are assumed to be 1010 $kg.m^{-3}$ and 8.9×10^{-4} *Pa.s*. The initial saturation of the leaching solution is assumed to be zero.

The leaching solution is applied to the heap from the top by a constant velocity of u_w = 10 *mm/hour*. On the sides, we assume no flow boundary condition for the leaching solution. The residual saturation of the leaching solution in the heap is assumed to be 0.15.

The two dimensional heap geometry is initially discretized into 782 elements and mesh adaptivity is applied based on the gradient of saturation. Therefore, as the liquid front moves through the heap, the mesh adapts itself to accurately resolve the flow dynamically with a minimum and maximum edge length of 0.5 and 3 meters respectively. The time step size is set to 10s. The simulation was performed on a personal computer with a 3.0GHz CPU. The run time was 6 hours.

Figure 4 shows the saturation distribution within the heap for *t*=33 hours and the surface mesh adapted to the gradient of saturation. It can be seen that the finer elements are formed in the region of heap where the saturation varies the most as expected.



(a)

(b)

Figure 4: The saturation distribution inside the heap geometry after *t*=33 hours (a) and the adapted mesh based on the gradient of saturation with a minimum and maximum edge length of 0.5 and 3 meters respectively (b).

Figure 5 shows the number of elements during the numerical simulation. The number of elements increases to around 3900 elements as the leaching solution front transits through the heap. After the breakthrough time of 81 hours, as the saturation front moves out of the heap, the number of elements reduces to 1400 elements.

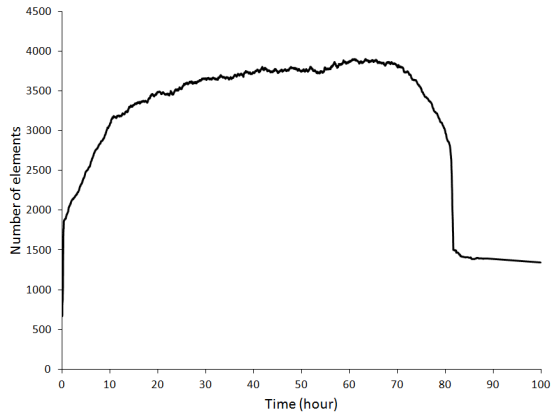


Figure 5: The number of finite elements for the heap leaching simulation versus time.

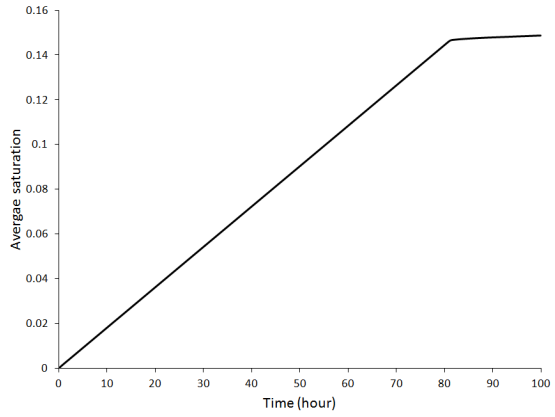


Figure 6: Average saturation of the leaching solution within the heap versus time.

Figure 6 shows the average saturation of the leaching solution within the heap versus time. The obtained numerical prediction exhibits the expected physical behaviour as the average saturation increases approximately linearly within the heap until the breakthrough time. Following the breakthrough of the leaching solution, the average saturation does not vary considerably as the process can be considered at a steady state.

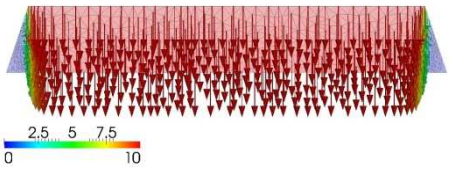


Figure 7: The vectors of the leaching solution velocity at the steady state.

Figure 7 shows the vectors of the leaching solution velocity at steady state. The velocity is almost constant and equal to the applied velocity at the top of the heap (10 *mm/hour*). This is in agreement with the expected physical behaviour at the steady state.

CONCLUSION

This paper presented a control volume finite element scheme for numerical modelling of multiphase flow in porous media. The governing equations are conservation of mass for each phase and Darcy's law. The developed numerical scheme employs an IMPES algorithm for the temporal discretization of the governing equations. Pressure is discretized spatially using a control volume finite element method. For saturation, a node centred control volume method is employed. The numerical scheme is implemented in Fluidity which contains advanced numerical features such as mesh adaptivity that can achieve high spatial accuracy for multiscale problems at low computational cost. The accuracy of the scheme is verified by comparing the numerical results against the quasi-analytical solution for the Buckley-Leverett problem. We showed the application of the method for the simulation of a heap leaching process. We simulated the two phase flow of air and the leaching solution within a two dimensional heap geometry and examined the use of mesh adaptivity to capture the evolving features of the flow. This illustrates the capability of the developed scheme for accurate numerical modelling of transport phenomena in porous media for large scale industrial applications. For future work, the developed scheme can be applied for simulation of non-uniform introduction of leaching solution in heterogeneous porous media in presence of chemical reaction occurring during the infiltration.

REFERENCES

AMCG, (2012), "Fluidity manual", *Department of Earth Science and Engineering, Imperial College London.*

AZIZ, K. And SETTARI, A. (1986), "Fundamentals of reservoir simulation", *Elsevier Applied Science Publishers, New York.*

BEAR, J., (1988), "Dynamics of fluids in porous media", *Dover publications, New York.*

BROOKS, R.H. and COREY, A.T., (1964), "Hydraulic properties of porous media". *Hydrology Papers, Colorado State University.*

BUCKLEY, S.E. and LEVERETT, M.C., (1942), "Mechanism of fluid displacement in sands", *TAIME*, **146**(1337), 107-116.

CARIAGA, E., CONCHA, F., and SEPÚLVEDA, M., (2005), "Flow through porous media with applications to heap leaching of copper ores", *Chemical Engineering Journal*, **111**(2), 151-165.

CARMAN, P.C., (1937), "Fluid flow through granular beds", *Transactions-Institution of Chemical Engineeres*, **15**,150-166.

DAVIES, D.R., WILSON, C.R. and KRAMER, S.C., (2011), "Fluidity: A fully unstructured anisotropic adaptive mesh computational modeling framework for geodynamics", *Geochemistry Geophysics Geosystems*, **12**(6), Q06001.

DULLIEN, F.A.L., (1992), "Porous media: fluid transport and pore structure", *volume 26. Academic Press, San Diego.*

GARCIA, X., AKANJI, L.T., BLUNT, M.J., MATTHAI, S.K. and LATHAM, J.P., (2009), "Numerical study of the effects of particle shape and polydispersity on permeability", *Physical Review E*, **80**(2), 021304.

HELMIG, R., (1997), "Multiphase flow and transport processes in the subsurface", *Springer-Verlag, Berlin.*

MCBRIDE, D., CROSS, M., CROFT, N., BENNETT, C.R. and GEBHARDT, J.E., (2006), "Computational modelling of variably saturated flow in porous media with complex three-dimensional geometries", *International Journal for Numerical Methods in Fluids*, **50**(9), 1085-1117.

MCBRIDE, D., GEBHARDT, J.E. and CROSS, M., (2012), "A comprehensive gold oxide heap leach model: Development and validation", *Hydrometallurgy*, **113-114**, 98-108.

MELLADO, M.E., CISTERNAS, L.A., and GÁLVEZ, E.D., (2009), "An analytical model approach to heap leaching", *Hydrometallurgy*, **95**(1-2), 33-38,

MUNOZ, J., RENGIFO, P. and VAUCLIN, M., (1997), "Acid leaching of copper in a saturated porous material: Parameter identification and experimental validation of a two dimensional transport model", *Journal of Contaminant Hydrology*, **27**(1), 1-24.

PAIN, C.C., MANSOORZADEH, S., DE OLIVEIRA, C.R.E. and GODDARD, A.J., (2001), "Numerical modelling of gas-solid fluidized beds using the two-fluid approach", *International Journal for Numerical Methods in Fluids*, **36**(1), 91-124.

PIGGOTT, M.D., GORMAN, G.J., PAIN, C.C., ALLISON, P.A., CANDY, A.S., MARTIN, B.T. and WELLS M.R., (2008), "A new computational framework for multi-scale ocean modelling based on adapting unstructured meshes", *International Journal for Numerical Methods in Fluids*, **56**(8), 1003-1015.

SHELDON, J.W., ZONDEK, B. and CARDWELL, W.T., (1959), "One dimensional, incompressible, noncapillary, two phase fluid flow in a porous medium", *TAIME*, **216**, 290-296.

STONE, H.L. and GARDER JR., A.O., (1961), "Analysis of gas-cap or dissolved-gas drive reservoirs", *SPE Journal*, **1**(2), 92-104.

