A New RNG-Based Two-Equation Model for Predicting Turbulent Gas-Particle Flows

A. Eghlimi[1], A. Kouzoubov[2] and C.A.J. Fletcher[1]

1. Center for Advanced Numerical Computation in Engineering and Science (CANCES)
The University of New South Wales, Sydney 2052, Australia
2. School of Physics and Electronic Systems Engineering, University of South Australia

ABSTRACT

A two-fluid Eulerian-Eulerian formulation has been implemented into the FLUENT CFD code to predict dilute gas-particle flows. The dynamic Renormalization group theory (RNG) based k-ε model (Orszag, 1993) were adopted for gas turbulence closure. Two models were applied to close particle turbulence. The turbulent gas-particle flow past a backward facing step which plays a significant role in benchmarking the performance of turbulence models for separated flows are simulated. The experimental study of Ruck and Makiola (1988) has been used to validate the models. The results are in good agreement with the experimental one.

1. INTRODUCTION

Dilute gas-particle flows are encountered in wide industrial applications such as combustion of pulverized coal, cyclone separators, classifiers, pneumatic conveying and electrostatic. Therefore numerical modelling of gas-particle flows, in which turbulence modelling plays a very important role, is of great importance. Elghobashi (1994) provides a comprehensive review of numerical models for dilute gas-particle flows.

There are basically two approaches in the computational modelling of the dispersed phase. The first one is the Lagrangian approach, which employs a statistical simulation where the trajectories of the particles are computed based on the equations of motion. In order to achieve statistically reliable results a relatively large number of particles should be tracked which makes this approach computationally expensive (Tsuji et al., 1985 and Chen and Pereira, 1995). More importantly, in this approach it is assumed that the presence of the particles has no influence on the under-lying turbulent motion. It has been shown (Moderrass et al., 1984) that the presence of even a small amount of particles can greatly change the intensity and eddy size of the fluid turbulence. In other words, the "one-way" coupling assumption that only fluid affects the particles is not complete and the model should take into account the "two-way" coupling effects.

On the other hand, the Eulerian approach treats both phases as two interpenetrating continua and solves for non-linearly coupled differential equations for each phase. In the Eulerian approach for the particulate phase many of the numerical models used the conventional two-equation k-ε model (Chen and Wood, 1985, Adeniji and Fashola, 1990 and Tu and Fletcher, 1994). In the present study, the renormalization group theory (RNG) based k-ε turbulence model (Orszag, 1993) is employed with our developing Eulerian model (Tu and Fletcher, 1994 and Eghlimi *et al.*, 1995).

Eulerian modelling often hinges on the modelling of particulate turbulent viscosity (Chen and Pereira, 1995). Aliod and Dopazo (1987) modelled the particulate turbulent viscosity, ν_p, using an expression similar to the modelling of gas viscosity. However, they had to solve another equation for the particle turbulent energy. Melville and Bray (1978) used the concept that turbulent viscosities of both phases are proportional to their turbulent energies which was restricted to axisymmetric jet flows. Based on the assumption that small particles should follow the fluid motion, Chen and Wood (1985) related the particulate turbulent viscosity, ν_p, to the gas turbulent viscosity in correlations that involved the

Stokes number. Rizk and Elghobashi (1989) in their formulation involved the turbulent kinetic energy and the relative velocities to relate the particulate turbulent viscosity to the gas turbulent viscosity.

In the present study the Chen and Wood (1985) model and another formulation that relates the particulate viscosity to the gas viscosity based on an exponential correlation are employed. The two models are then tested against the experimental study of Ruck and Makiola (1988) for gas-particle flow over a backward facing step.

2. GOVERNING EQUATIONS

In this section, we present the assumptions made and the forms of the governing equations adopted for predicting turbulent flyash flows. The main assumptions employed in the present study are as follows:

1. The particulate phase is dilute but continuous and consists of mono disperse spherical particles; the fluid phase is Newtonian; and the physical properties of each phase are constant.

2. Particle-particle interactions are neglected and fluid-particle two-way interaction is allowed; forces due to the gas pressure gradient, Magnus forces due to particle rotation and Saffman forces due to the viscous shear-rate on the particulate phase are negligible.

3. The mean flow is steady, three-dimensional, incompressible and isothermal.

4. The third order correlations involving fluctuations in the particulate phase concentration are negligible.

For the derivation of the governing equations in the Eulerian formulation one is referred to the work done by Drew (1983). The summary of these equations is presented here. Hinze (1972), Elghobashi (1994) and Tu and Fletcher (1995) discuss the criteria for which the above assumptions as well as the continuum approach conditions are valid.

2.1 Gas Phase

Governing equations in the Cartesian form for the mean turbulent gas flow are obtained by applying the Reynolds decomposition and time averaging the instantaneous continuity and momentum equations:

$$\frac{\partial}{\partial x_i}(\rho\, u_i) = 0 \qquad (1)$$

$$\frac{\partial}{\partial x_j}(\rho\, u_i u_j) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\rho \nu_l \frac{\partial}{\partial x_j} u_i\right) - \frac{\partial}{\partial x_j}\left(\rho\, \overline{u_i' u_j'}\right) - F_{Di} \qquad (2)$$

where ρ, u, u' and P are bulk density, mean velocity, fluctuating velocity and mean pressure of the gas phase, respectively. ν_l is the gas laminar viscosity and F_{Di} is aerodynamic drag force due to the slip velocity between two phases which is given in section 2.2.

2.2 Particulate Phase

After Reynolds averaging, the steady form of the governing equations for the particulate phase is:

$$\frac{\partial}{\partial x_i}(\rho_p u_{p_i}) = -\frac{\partial}{\partial x_i}(\overline{\rho_p' u_{p_i}'}) \qquad (3)$$

$$\frac{\partial}{\partial x_j}(\rho_p u_{p_i} u_{p_j}) = -\frac{\partial}{\partial x_j}\left(\rho_p \overline{u_{p_i}' u_{p_j}'} + u_p^j \overline{\rho_p' u_{p_i}'} + u_{p_i} \overline{\rho_p' u_{p_j}'}\right) + F_{Gi} + F_{Di} + F_{WMi} \qquad (4)$$

where ρ_p and ρ'_p are the mean and fluctuating mass of particles per unit volume of mixture (or 'bulk density' of the particulate phase where $\rho_p = \alpha_p \rho_s$, α_p is the volume fraction of the particulate phase and ρ_s is particle material density). It may be noted that due to the dilute particulate phase assumption, the multiplication of each term in the equations for the gas phase by $(1-\alpha_p)$ is replaced by 1. u_p and u_p' are the mean and fluctuating velocity of the particulate phase, respectively.

In equation (4), there are three additional terms representing the gravity force, F_{Gi}, aerodynamic drag force, F_{Di}, respectively. The wall-momentum transfer force due to the particle-wall collision, F_{WMi}, has been derived from impulsive equations in the normal and tangential directions (Tu and Fletcher, 1995). The gravity force is $F_G = \rho_p g$, where g is the gravitational acceleration. The drag force F_D due to the slip velocity of two phases is defined by:

$$F_{Di} = \rho_p \frac{f\,(u_i - u_{pi})}{\tau_p} \tag{5}$$

Schuh *et al.* (1989) suggested the value of f based on particulate Reynolds number:

$$f = \begin{cases} 1+0.15Re_p^{0.687} & 0<Re_p \le 200 \\ 0.914Re_p^{0.282}+0.0135Re_p & 200<Re_p \le 2500 \\ 0.0167Re_p & 2500<Re_p \end{cases} \tag{6}$$

Re_p is given:

$$Re_p = \frac{|u_i - u_{p_i}|\, D_p}{\nu_l} \tag{7}$$

particle response or relaxation time is:

$$\tau_p = \frac{\rho_s D_p^2}{18\rho\,\nu_l} \tag{8}$$

where D_p is the diameter of the particle. For a dilute gas-particle flow, the viscous and pressure terms in the particulate phase momentum equations are neglected.

2.3 Turbulence Models

Averaging process introduces more unknowns than the number of equations. The second order correlations $\overline{u_g'^i u_g'^j}$, $\overline{u_g'^i u_p'^i}$, $\overline{u_p'^i u_p'^j}$, $\overline{\rho_p' u_p'}$, which are turbulence fluxes of momentum and mass, require modelling.

A significant part of most turbulence models turned out to be based on the oldest model in modeling turbulence proposed by Boussinesq (1877). In his eddy-viscosity concept the turbulent stresses are proportional to the mean-velocity gradients:

$$-\overline{u_i u_j} = \nu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3}k\,\delta_{ij} \tag{9}$$

where δ_{ij} is the Kronecker delta and k is the kinetic energy of the fluctuating motion and ν_t is the turbulent or eddy viscosity for the gas phase which is evaluated by (ν_{eff} - ν_l), where the effective viscosity ν_{eff} is computed by:

$$\nu_{eff} = \nu_l\left(1 + \sqrt{\frac{C_\mu}{\nu_l}}\frac{k}{\sqrt{\varepsilon}}\right)^2 \tag{10}$$

The kinetic energy of the turbulence, k and its dissipation rate, ε are governed by separate transport equations. The RNG-based k-ε turbulence model (Orszag, 1993) contains very few empirically adjustable parameters and is therefore applicable to a wide range of flow situations. The k and ε transport equations are modified by the consideration of particulate turbulence modulation as:

$$\frac{\partial}{\partial x_i}(\rho_g u_i k) = \frac{\partial}{\partial x_i}\left(\alpha \rho_g \nu_t \frac{\partial k}{\partial x_i}\right) + P_k - \rho_g \varepsilon + S_k \tag{11}$$

$$\frac{\partial}{\partial x_i}(\rho_g u_i \varepsilon) = \frac{\partial}{\partial x_i}\left(\alpha \rho_g \nu_t \frac{\partial \varepsilon}{\partial x_i}\right) + \frac{\varepsilon}{k}(C_{\varepsilon 1}P_k - C_{\varepsilon 2}\rho_g \varepsilon) - \rho_g R + S_\varepsilon \tag{12}$$

where α is an inverse Prandtl number which may be obtained from the following equation:

$$\left|\frac{\alpha - 1.3929}{\alpha_0 - 1.3929}\right|^{0.6321}\left|\frac{\alpha + 2.3929}{\alpha_0 + 2.3929}\right|^{0.3679} = \frac{\nu_l}{\nu_{eff}} \tag{13}$$

where α_0=1. The turbulence production P_k is evaluated by

$$P_k = \rho_g \nu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\left(\frac{\partial U_i}{\partial x_j}\right) \tag{14}$$

The rate of strain term R in the ε-equation is expressed as

$$R = \frac{C_\mu \eta^3 (1 - \eta/\eta_0)}{1 + \beta\eta^3}\frac{\varepsilon^2}{k} \tag{15}$$

and

$$\eta = \frac{k}{\varepsilon}(2S_{ij}^2)^{1/2}, \qquad S_{ij} = \frac{1}{2}\left(\frac{\partial u_g^i}{\partial x_j} + \frac{\partial u_g^j}{\partial x_i}\right) \tag{16}$$

where the constants are given based on table 1.

Table 1 RNG-Based k-ε Model Constants

β	η_0	C_μ	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$
0.015	4.38	0.0845	1.42	1.68

Hetsroni (1989) indicated that the presence of particles with diameter less than 200 μm absorbs some of the turbulent energy present in the carrier gas flow. This is typically modelled with extra dissipation terms S_k and S_ε in the k and ε transport equations, respectively. For the confined two-phase flow, the effects of the particulate phase on the turbulence structure of the gas phase $(-\overline{u_i' F_{D_i}'})$ are given by Tu and Fletcher (1994):

$$S_k = -2k(\rho_p / t_p)[1-\exp(-B_k t_p / t_L)] \quad (17)$$

$$S_\varepsilon = -2\varepsilon(\rho_p / t_p)[1-\exp(-B_\varepsilon t_p / t_L)] \quad (18)$$

for the k and ε equations respectively, where B_k= 0.09, B_ε= 0.4 and t_L=k/ε. Chen and Wood (1985) and Adeniji and Fashola (1990) used a gradient hypothesis to yield the second-order correlation terms in governing equations of the particulate phase:

$$-\overline{\rho_p' u_{p_i}'} = \Im_p \frac{\partial \rho_p}{\partial x_i}$$
$$-\overline{u_{p\,i}' u_{p\,i}'} = \nu_p \left(\frac{\partial u_{p_i}}{\partial x_j} + \frac{\partial u_{p_i}}{\partial x_i}\right) \quad (19)$$

where the particulate turbulent diffusivities $\Im_p$ and ν_p are related to the turbulent viscosity of the gas phase ν_t by:

$$\Im_p = \frac{\nu_p}{S_c} \quad ; \quad \nu_p = K_p \nu_t \quad (20)$$

and S_C is the turbulent Schmidt number taken to be 0.7. K_p is a weight factor accounting for the particle inertia which is given by either **model A**:

$$K_p = \max[K_d, 1/(1+St)] \quad (21)$$

or **model B:**

$$K_p = \max[K_d, \frac{1}{e^{0.0825St}} \nu_t)] \quad (22)$$

where K_d is a numerical dissipation and St is the turbulent Stokes number= τ_p/τ_e. Here, the turbulent eddy characteristic time follows the work of Adeniji-Fashola (1990) and Chen and Wood (1985) for the confined two-phase flow, τ_e=0.125k/ε. Thus, K_p is reflecting the transfer of turbulent energy to the particulate phase due to the particle inertia. Near the wall, this value becomes very small (less than 0.001) and K_d is taken to be 0.01 to avoid numerical instability. Model A (Equation 21) and model B (Equation 22) will be validated against experimental data.

3. NUMERICAL PROCEDURE

User defined subroutine option in FLUENT is used to implement the particulate concentration and velocity components as additional user defined scalars. An additional equation is then solved for each scalar. The required diffusion coefficient are derived from the momentum equations and the turbulence model. Source terms for particulate momentum scalar equations are computed by integrating over a control volume.

The power law interpolation scheme is used to compute the source terms. At the inlet boundary the particulate phase velocity is taken to be the same as the gas velocity. The concentration of the particulate phase are set to be uniform at the inlet. At the outlet the zero streamwise gradients are used for all variables. The boundary conditions at the wall for the particulate phase are based on the model of Tu and Fletcher (1995).

4. RESULTS AND DISCUSSIONS

In this section, the validation of the numerical computation using the two particulate turbulent viscosity models are presented. The experimental data of Ruck and Makiola (1988) is used. The particle size is taken to be 70 μm.

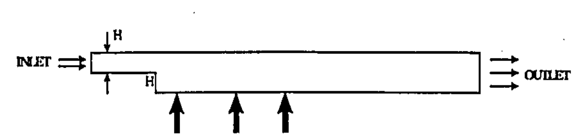

Fig 1. Ruck and Makiola backward-facing step

Figure 1, illustrates the backward facing step employed in the present study with step height, H, and the inlet height of 25 mm each. The distance behind the step is taken to be 25H. The reason for having this lengthy distance is to avoid any influence of the outlet boundary condition on the gas and particulate flow field.

At three different locations behind the step (illustrated in Figure 1) the numerical results are compared with the experiments. The Re_H is taken to be 64000. The material density of the particles, ρ_s, is taken to be 1500kg/m^3. A 200x60 non-uniform grid is used.

The RNG-based k-ε turbulence model for the gas-phase in the backward-facing step flows has shown to perform better than the k-ε model (Eghlimi, et al., 1995). This model is used here for the closure of the gas-phase. Figure 2 illustrates the mean velocity of the particulate phase predicted by Chen and Wood correlation (model A) and the exponential correlation (model B).

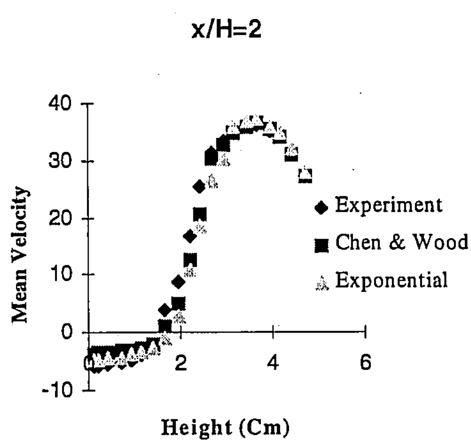

Fig 2 Mean velocity profile for 70 μm particles

As we can see, at the back of the step (Height<2.5cm) where the anisotropy exists both models over predict the mean velocity. On the other hand, further down the stream both models seem to predict the particulate mean velocity very close to the experimental values.

Figures 3 and 4 illustrate the particulate mean velocities at x=6H and x=9H.

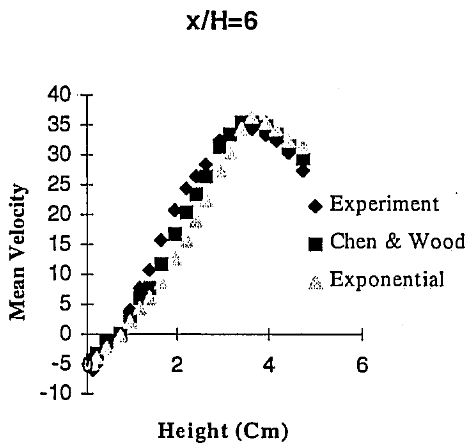

Fig 3 Mean velocity profile for 70 μm particles

It can be seen in Figures 3 and 4 that model a and model B give fairly good agreement with the experimental data for this dilute gas-particle flow. The exponential model predicts more accurately in the region closer to the upper wall.

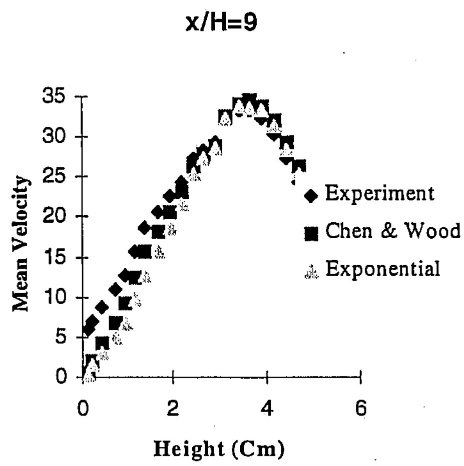

Fig 4 Mean velocity profile for 70 μm particles

The isotropic nature of the turbulent diffusivities in models A and B is open to criticism (Tu and Fletcher, 1995). However the influence of this assumption is rather small for the dilute gas-particle flow considered in the present study.

5. CONCLUSIONS

Two particulate turbulence models have been studied to predict turbulent gas-particle flow in a backward-facing step (Ruck and Makiola, 1988). Both models illustrated good agreement with the experimental data. In the future, the models will be validated against other experimental data such as: different particle sizes, r.m.s. velocity profile, maximum positive and negative velocities, etc. The model will then be extended to more complex closure correlations.

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