Computation of the oscillation of free surfaces and the effect of electromagnetic fields

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ABSTRACT

Oscillating free surfaces or interfaces occur in many metallurgical processing operations. One example is the Hall cell used in the production of aluminum where the interface between the molten salt electrolyte and aluminum is susceptible to surface waves. A second important example is the free surface of the metal in the casting of aluminum, or the flux-metal interface in the continuous casting of steel. Surface waves in these casting operations can cause periodic irregularities in the surface of the cast metal. The paper examines the numerical solution of the governing equations for a free surface or liquid/liquid interface. 2D solutions for the behaviour of an unconstrained (in the vertical direction) surface are presented, together with the results of calculations where electromagnetic forces act on the melt through the effects of an imposed electromagnetic field. The last calculations are self-consistent in that the effect of the surface shape on the electromagnetic field is allowed for.

1. INTRODUCTION AND OBJECTIVE

Over one million tons of aluminum are solidified each year by electromagnetic casting (EMC). As illustrated in Figure 1, the technology is one where solidification proceeds without any contact between the molten metal and a mold. Instead, the liquid aluminum is supported from below by the metal that has already solidified, and on the sides by electromagnetic forces. These forces arise from the interaction between currents induced in the aluminum and a magnetic field. The field is created by alternating current (a few kiloAmps at a few kHz) flowing in an inductor surrounding the aluminum pool. A "screen" or "shield" is partly interposed between the inductor and metal so as to modify the magnetic field (Evans, 1995).

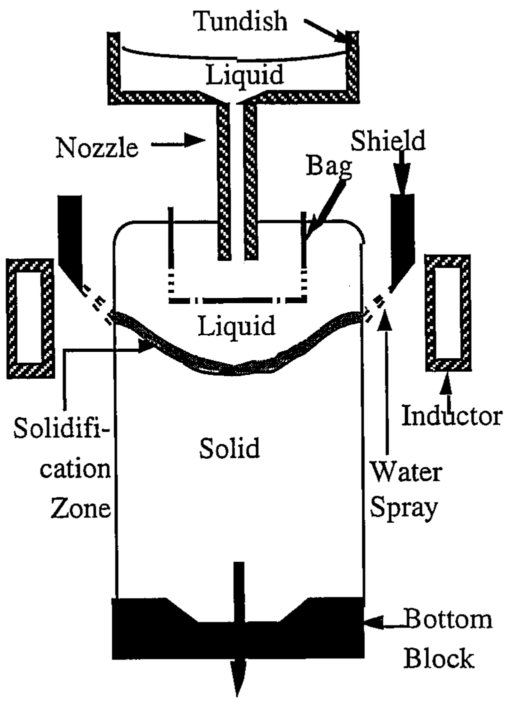

Figure 1. Schematic cross-sectional view of an electromagnetic caster.

One of the principal advantages of EMC is that the resulting ingot usually has a very smooth surface. This is in contrast to the ingots produced by the rival semi-continuous casting technology "direct chill" (DC) casting. In DC casting solidification occurs with the aluminum in contact with an actual mold (water-cooled copper) and the ingot must be machined ("scalped") after casting to yield a surface smooth enough for the subsequent rolling operations. The loss of production and expense associated with scalping is significant and their avoidance may justify the extra capital and operating cost of EMC. Such justification is lost, however, if the EMC ingots require scalping and the smoothness of the surface of the metal in EMC is therefore of interest.

One irregularity frequently seen on EMC ingots is a surface wave, frequently extending around the whole ingot with the direction of the wave vector vertical. A ready explanation for this wave is that it is caused by oscillations in the surface of the liquid pool. Because the liquid is confined only by "soft" electromagnetic forces, outward and inward bowing of the liquid surface at the solidification line causes such waves on the surface of the solidified metal. There are obvious sources of disturbance of the liquid metal pool (jerky descent of the hydraulically supported bottom block below the ingot, or building vibration) but such sources have been largely eliminated. In a recent paper Deepak and Evans (1995) carried out a linear perturbation analysis which suggested that the electromagnetic field itself could destabilize the liquid metal surface above a critical magnetic field strength. That strength was comparable with that encountered in actual casters.

The objective of the research described in this paper was to further examine the stability of a liquid metal pool in an alternating magnetic field by a combination of experiment and numerical calculations. The results include the more general case where there is no magnetic field and the decay of surface oscillations is of interest.

2. NUMERICAL STUDIES

2.1 Approach

For calculation of the dynamic behaviour of the metal pool in the presence of an electromagnetic field, solution of Maxwell's equations (coupled with Ohm's law) for the electromagnetic field, and the Navier-Stokes/continuity equations for the flow are required. The solution for the field and flow must be closely coupled because changes to the shape of the free surface cause changes in the electromagnetic field which then alter the forces on the melt and its surface shape and flow.

The present calculations treat a cylindrical melt in an axisymmetric magnetic field (see Figure 2), undergoing oscillation of its upper free surface so that the surface remains axi-symmetric. Space limitations preclude a full description of how Maxwell's equations were solved but that description is found in the thesis on which this paper is based (Kageyama, 1995). The equations were solved in terms of a vector potential using finite differences on a body-fitted co-ordinate system. A boundary condition for the vector potential was first computed on an imaginary boundary somewhat outside the melt by the "method of inductances" (solution of Maxwell's equations in integral form). The imaginary boundary was positioned so that melt surface perturbations did not alter the vector potential on the boundary significantly.

The Navier-Stokes and continuity equations were solved in their instantaneous (rather than time-averaged) form following the approach of Kawamura et al. (1986). Free-slip boundary conditions were imposed on the top surface of the melt with no-slip boundary conditions imposed on the melt-container interfaces at the bottom and sides (simulating the conditions of the experiment, rather than those of EMC). Pressure in the liquid just below the free surface was adjusted to allow for surface tension. The height of the free surface (h) was computed from the horizontal velocity (u) and vertical velocity (w) at the free surface by

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} = w \qquad (1)$$

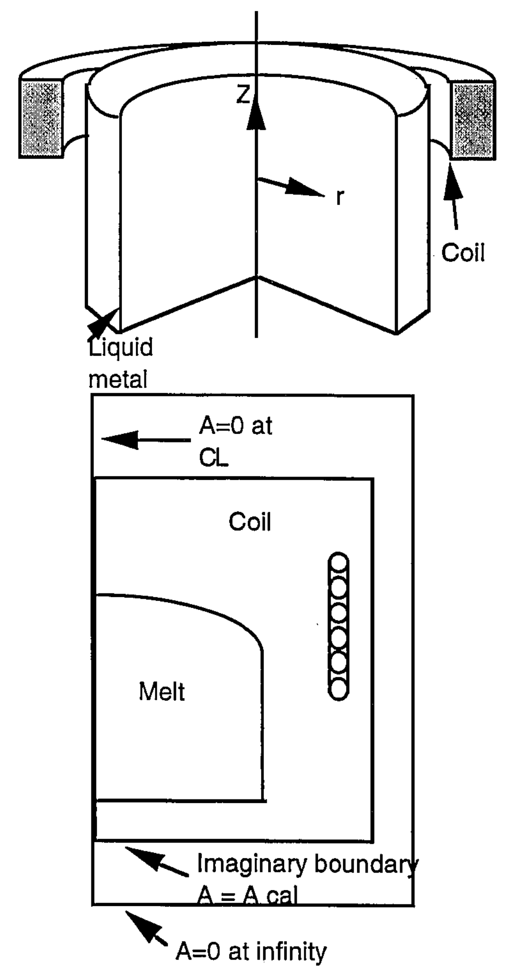

Figure 2: Geometry of experiment and mathematical model; boundary condition employed for vector potential

Various finite difference schemes (in the moving, body-fitted co-ordinate system) were used to solve the fluid flow equations. The mesh used was 50 by 25 (the latter being the vertical direction). As reported below, all schemes suffered from numerical damping and this was revealed by testing each by computing the oscillation of an inviscid fluid in the absence of an electromagnetic field. The Navier-Stokes equations were solved explicitly with a Poisson equation for pressure arising from the continuity equation. The pressure-smoothing technique of Sotiropoulos and Abdallah (1991) was employed. Equation (1) was solved in explicit finite difference form using upwind differences for the convective term. Noise was minimized by using a filtering technique of Miyata and co-workers (1987).

2.2 Numerical results

The oscillation of the free surface of a cylindrical melt is a classical problem and the eigenfrequencies can be calculated from (Lamb, 1932):

$$\left(2\pi\Omega_{m,n}\right)^2 = \left(\rho g + \sigma_s\left(\frac{\lambda_n}{r}\right)^2\right)\frac{\lambda_n}{\rho r}\,tanh\left(\lambda_n\frac{H}{r}\right)$$

where $\Omega_{m,n}$ is the eigenfrequency, m and n are the azimuthal (circumferential) and radial wave numbers, respectively, r and H are the radius and depth of the pool, σ_s the surface tension, ρ the liquid density and λ_n the nth zero of the first derivative of the mth order Bessel function of the first kind. These eigenfrequencies are given in Table I for a mercury pool of 25 mm depth and 200 mm diameter that was the subject of the experimental and numerical studies.

Table I
Eigenfrequencies of the mercury pool

m	n=1	2	3	4
0	2.67	4.09	5.10	5.93
1	1.41	3.41	4.61	5.52
2	2.21	3.97	5.04	5.89
3	2.87	4.43	5.42	6.26
4	3.40	4.84	5.77	6.59
5	3.86	5.19	6.10	6.92
6	4.26	5.52	6.42	7.24

To test the numerical scheme, and its variants, the oscillation of a free surface which was initially deformed downward by 2.5 mm, to yield the shape of an inverted cone, was computed with the electro-magnetic field and viscosity set to zero. Physically, such a pool should oscillate indefinitely with constant amplitude. Figure 3 shows the computed oscillations at the pool center for a first order upwind scheme. The oscillations have a clearly defined frequency of 2.7 Hz which matches well with the first eigenfrequency for m=0 (i.e., a radial wave) in Table I. Unfortunately the oscillations are decaying so this scheme displays numerical damping. All numerical schemes tried at the time of writing display numerical damping, some worse than that seen in Fig. 3. The reduction ratios for the amplitude at 10 seconds are given in Table II. By this criterion, the third order upwind scheme is the most accurate. The third and fourth order schemes in Table II are special cases of the equation provided by Kawamura et al. (1995). That equation provides a parameter (α) which may be set to yield various schemes (e.g., α=1 is equivalent to third order schemes). Attempts to use α=6 or 12 did not eliminate numerical damping. Nor was the damping eliminated by turning off Miyata's noise filtering technique. [Similar numerical damping was found when the commercial code FLUENT 4.32 was employed on this problem with first or second order upwind differencing.]

Table II
Reduction ratio for various numerical schemes (wave amplitude at 10 sec./initial wave amplitude)

Scheme	Reduction Ratio
First order	0.795
Second order	0.829
Third order	0.87
Fourth order	0.822
α=6	0.757
α=12	0.658

Calculations were then performed for the case of an applied electromagnetic field (corresponding to the experimental case with an inductor current of 400 Amps) and physical properties set to those of mercury. As will be seen in the next section, the actual metal oscillates with frequencies close to the classical eigenfrequencies and amplitude that appears steady. The results of applying the third order upwind scheme are seen in Figure 4. In these (and subsequent) calculations the current was turned on over a three-second interval. The point in question is at the center of the top surface and is displaced upward by the pinching effect of the electromagnetic forces. The computed amplitude is increasing with time and, by the end of 20 seconds is, at ± 4 mm greatly in excess of the experimental value of roughly ± 0.2 mm. It is concluded that the third order upwind scheme is displaying numerical instability in these calculations although the 2.7-Hz classical eigenfrequency is displayed by the results. The fourth order upwind scheme has similar difficulties. In contrast, Figure 5 shows a steady (but modulated) oscillation of the surface with an amplitude close to the experimental value. These computations were carried out with the Kawamura scheme and α=6.

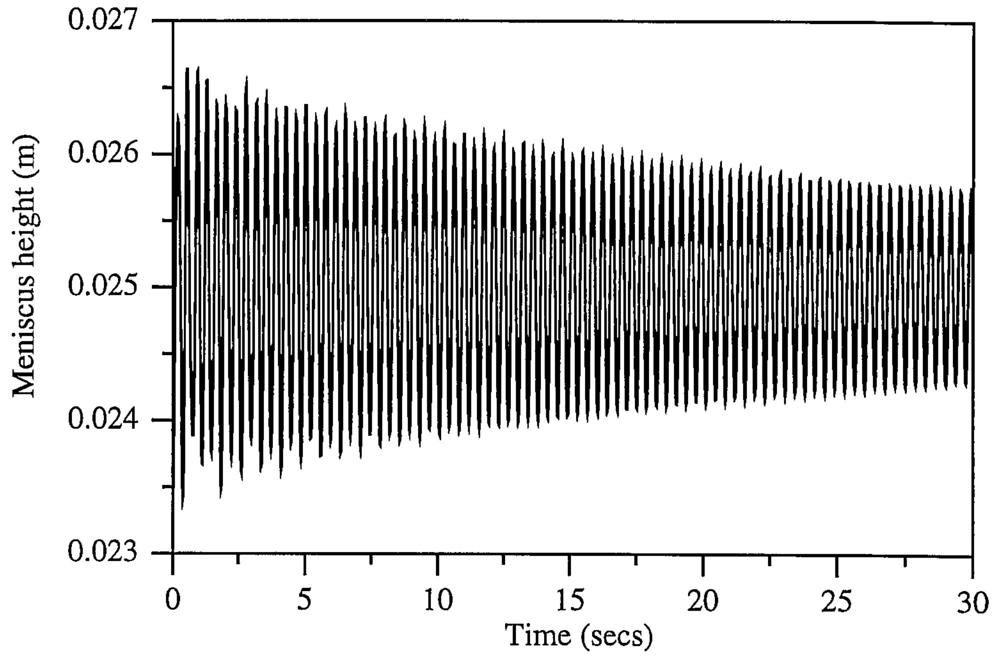

Figure 3
Computed oscillations of center point of pool surface for no electromagnetic field and inviscid liquid (first order upwind scheme).

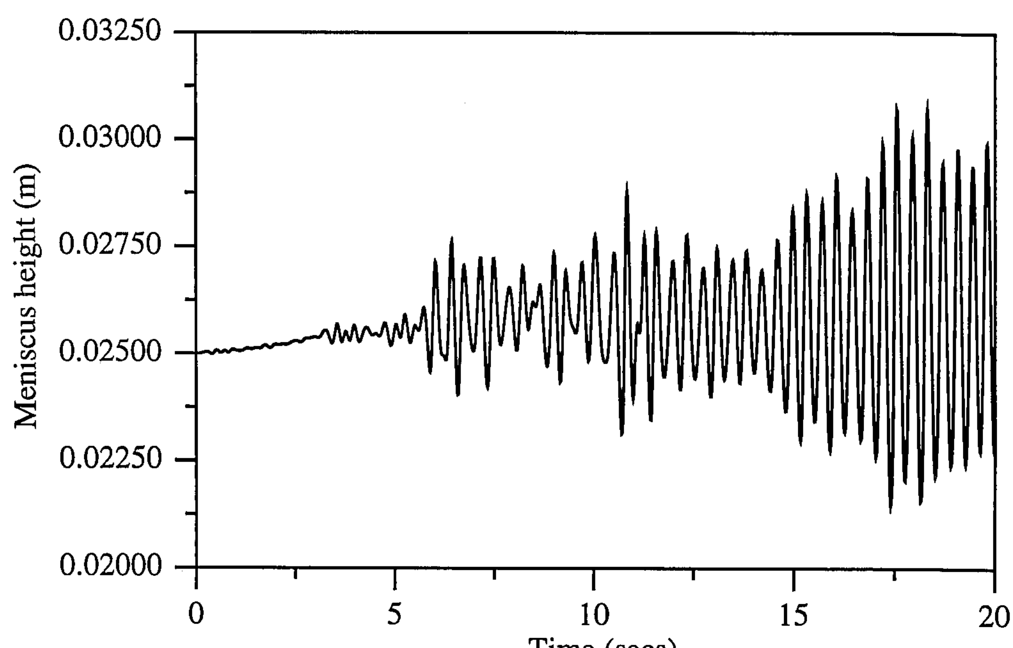

Figure 4
Computed behaviour of center point of pool surface for properties of mercury and with field turned on over first three seconds and thereafter (third order upwind scheme).

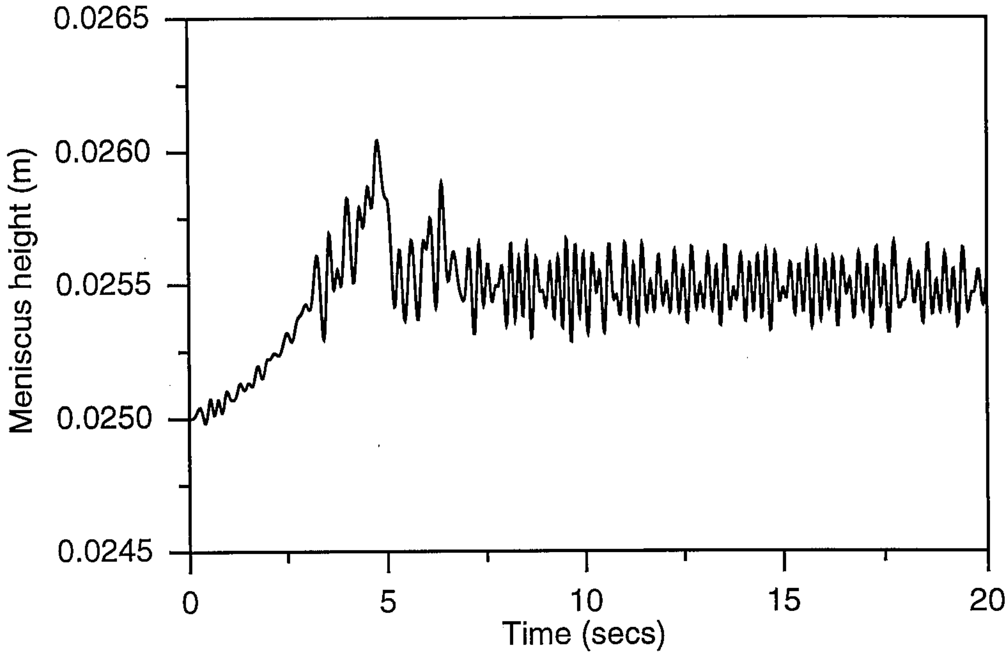

Figure 5
As in Figure 4 but with the scheme of Kawamura, Takami and Kuwabara (1986) and α=6.

3. EXPERIMENTAL MEASUREMENTS AND RESULTS

Figure 6 depicts the mercury pool and laser vibrometer used in the experiment. The mercury is surrounded by an inductor driven by a 3-kHz power supply. Details of the apparatus can be found in the thesis of Kageyama (1995), as can additional experimental results.

With no magnetic field applied the meniscus showed oscillation of $\pm$0.1 mm with a frequency (2.7 Hz) corresponding to the first radial mode of Table I. These oscillations were probably due to building vibration or disturbance caused by flow of water in the cooling system. Oscillations at the center of the pool are seen in Figure 7 for the case where the inductor current was set to 400 A. The oscillations are seen to have increased to $\pm$0.2 mm and display some modulation. The power spectrum of the surface oscillations can be seen in Figure 8 and again the classical first radial mode oscillation around 2.7 Hz is dominant. Oscillations increase yet further on increasing the current, reaching approximately $\pm$0.5 mm at 500 A.

4. CONCLUDING REMARKS

The flow and oscillation of a metallic pool in an electromagnetic field have been examined. As a preliminary part of the study the behaviour of an inviscid cylindrical pool with the field turned off has been computed using various differencing schemes. All schemes and a commercial code showed, to greater or lesser degree, numerical damping. In the presence of the magnetic field one of the schemes showed stability and yielded an oscillation amplitude close to that determined in the experimental part of the investigation. In all cases the dominant frequency was the eigenfrequency anticipated from the classical theory of oscillation of a metal pool. It is concluded that an alternating magnetic field can destabilize a molten metal pool. A tentative explanation for the oscillations is that the electromagnetic field causes turbulent flow in the melt and the turbulence then excites the eigenfrequencies of the melt.

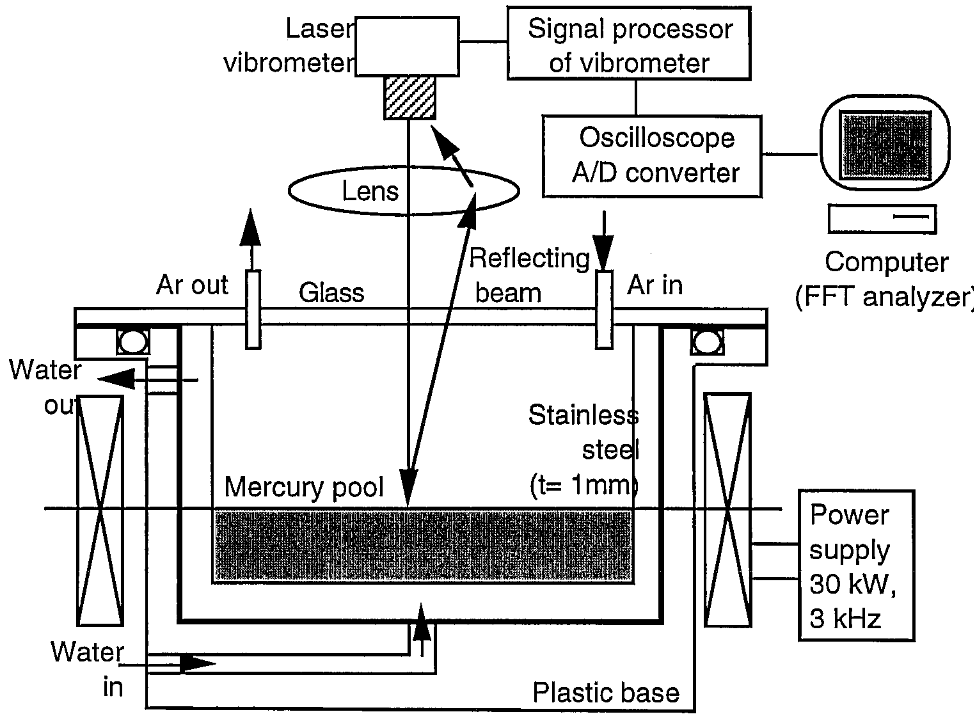

Figure 6
Experimental apparatus

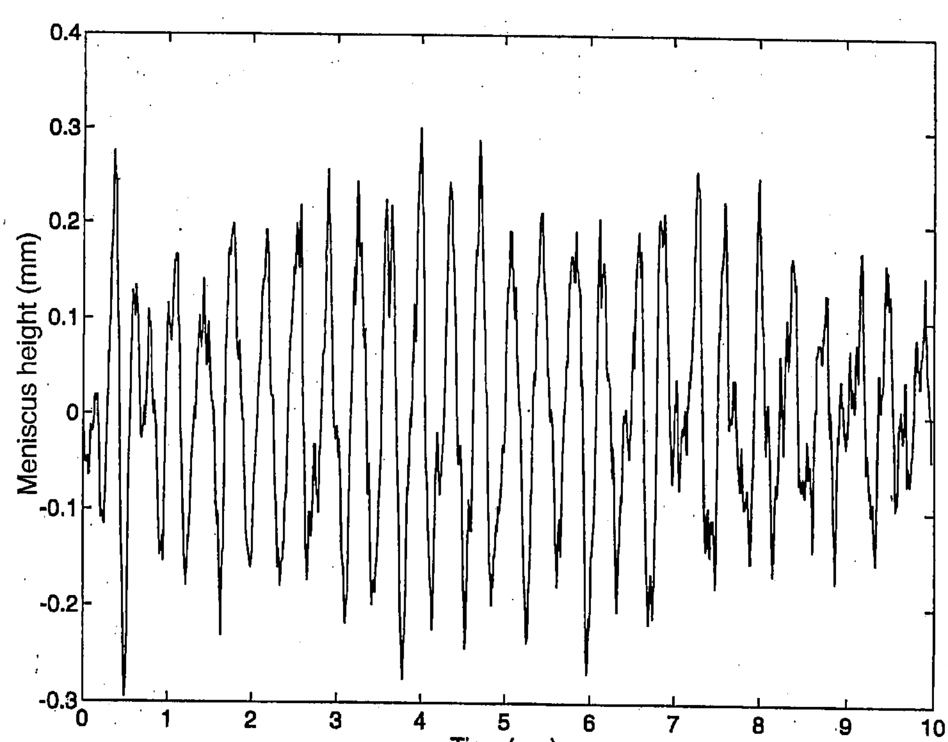

Figure 7
Measured oscillations at center of mercury pool for an inductor current of 400 Amps.

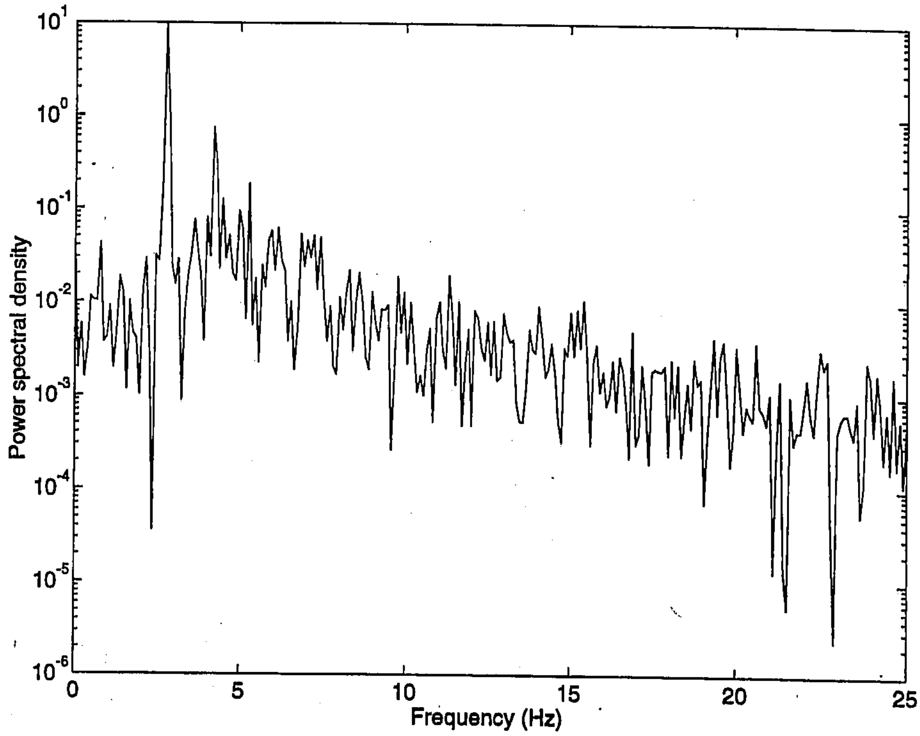

Figure 8
Fast Fourier transform of the oscillations of Figure 7.

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