Second International Conference on CFD in the Minerals and Process Industries
CSIRO, Melbourne, Australia
6-8 December 1999

HYSTERESIS IN THE OPEN PIPE FLOW WITH VORTEX BREAKDOWN

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ABSTRACT

The region in swirl/Reynolds number space where hysteresis is evident in the open pipe flow has been investigated, with the aim of determining whether it is possible to cause a jump from one state (without vortex breakdown) to another (with vortex breakdown) at the same swirl and Reynolds number. This initial study has showed that by introducing a large perturbation in the form of a transient swirl increase it is possible to generate a breakdown bubble, but this bubble disappears once the perturbation has propagated through the pipe.

NOMENCLATURE

Re Reynolds number
u Axial velocity
v Radial velocity
w Azimuthal velocity
Ω Swirl number

INTRODUCTION

Vortex breakdown has been observed in a number of swirling flows, including the leading edge vortex generated above delta wings, in open pipes with swirling inflows, and in the torsionally driven cylinder. Breakdown has also been observed in cyclonic separators and is used as a flame holder in combustion chambers.

Vortex breakdown is characterised by a stagnation of the vortex core, followed by an expanding structure which can take on a number of forms. The most prevalent forms include the axisymmetric bubble and spiral. The spiral mode of vortex breakdown has been observed in the exhaust regions of cyclone chambers, and can result in a vortex "whistle'' (Gupta *et al.,* 1984). For higher swirl, the vortex precession downstream of vortex breakdown can extend into the cyclone. Vortex core precession (the spiral mode of vortex breakdown) has also been observed in long cyclone dust separators for which length/diameter is greater than 4. This precession hampers the dust removal process.

Since its discovery a number of studies have focused on determining a cause for vortex breakdown, and some theories have been proposed to explain the phenomenon. In this work we focus on the explanation proposed by Darmofal and Murman (1994). In their study the wave trapping nature of vortex breakdown was investigated. This theory proposes a transition between two conjugate states, one called supercritical, upstream of breakdown, and the other called subcritical, downstream of breakdown. The idea of two conjugate states was due to Benjamin (1962). This theory stated that vortex breakdown could be described as the transition between these two states where excess "flow force'' could be dissipated. Darmofal and Murman's (1994) analysis concentrated on the wave trapping property of the vortex as its status changed from supercritical, and hence incapable of sustaining upstream travelling waves, to subcritical, and capable of sustaining such upstream travelling waves.

The work presented here primarily examines the hysteretic nature of vortex breakdown. Beran and Culick (1992) studied the hysteresis inherent in an open pipe flow with swirling inflow. It was observed that for a sufficiently large Reynolds number, the specification of a particular swirl in a certain range could result in more than one flow state, depending on how that swirl level was approached. Figure 1 represents this phenomenon qualitatively.

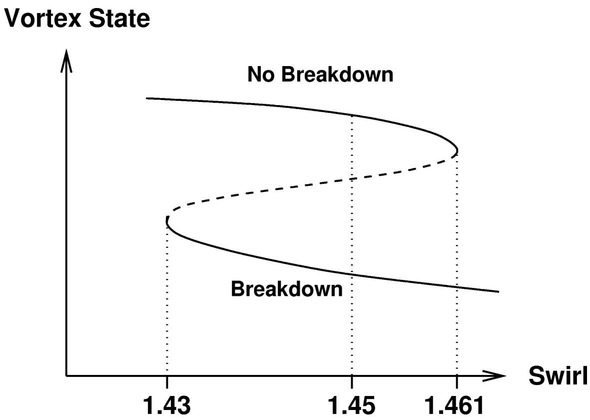

Figure 1: Hysteresis in the open pipe, due to Beran and Culick (1992).

Lopez (1994) considered the same pipe geometry as Beran and Culick (1992), and also observed the hysteresis found in their study – the specification of inlet boundary conditions alone was not sufficient to determine the nature of the flow downstream for a certain *Re*/Ω combination. Shtern and Hussain (1999) give a general discussion of fold catastrophes and hysteresis in swirling flows.

In the present study the hysteretic nature of vortex breakdown in the open pipe will be investigated. The hysteresis observed by Beran and Culick (1992) will be verified and a further study will be conducted into the possibility of jumping between states at a common swirl.

PROBLEM FORMULATION

This study will focus on the region in the swirl-Reynolds number parameter space where hysteresis occurs in the open pipe. An initial investigation has determined that for a Reynolds number of Re=1000, hysteresis is observed in the range 1.43<Ω<1.461, where Ω is the swirl parameter defined in equations (1). This compares with Beran and Culick's (1992) range of 1.465<Ω<1.505 for the same Reynolds number. The relative range shift may be due to the slightly different geometries considered - both of the pipes are identical apart from a converging outlet in our study. The two ranges are of comparable size.

The pipe geometry is defined in Darmofal and Murman (1994). It is comprised of a converging, then diverging inlet section, a straight test section, and a converging outlet. Figure 2 shows the pipe geometry and grid.

The inlet velocity profile is the q-vortex, defined in Leibovich (1983):

$$\begin{aligned} u_0(r) &= 1 + \Delta u \exp[-r^2] \\ v_0(r) &= 0 \\ w_0(r) &= \Omega[1 - \exp[-r^2]]/r \end{aligned} \qquad (1)$$

where $u_0(r)$ is the axial velocity, $v_0(r)$ the radial velocity, and $w_0(r)$ the azimuthal velocity at the inlet. This profile is identical to that used in Beran and Culick (1992), but Ω replaces V, the 'vortex strength', used in their study.

We begin with a steady solution at Ω=1.43, and steadily increase the swirl, keeping the axial velocity constant, until breakdown is observed at Ω=1.461. This process defines the limit on the upper branch for the no breakdown state. Next the swirl is reduced, so that the flow travels along the lower branch in figure 1, until the breakdown bubble disappears at Ω=1.43. This defines the limit point on the lower branch. This second limit point is less well defined - the bubble seems to be more persistent once it has evolved, and the flow takes a long time to return to the original pre-breakdown state.

The main part of the study involves starting with the pre-breakdown flow in the hysteresis region, then perturbing the solution to determine whether a transition to the breakdown state could be triggered. Tests with small perturbations of around ΔΩ = 0.01 resulted in little change in the flow. A doubling of the swirl was required in order to trip breakdown. The increase in Ω is maintained for 10 timesteps, after which time the swirl was returned to the initial Ω=1.45. This is in contrast to Darmofal and Murman's (1994) study, where the swirl was increased, then maintained at the new level, and the flow evolution observed.

In order to visualise clearly the behaviour of the system as it adjusts to the change in swirl, we plot two dependent variables - the change in the azimuthal vorticity: $\varepsilon_{pert} = \varepsilon_{timestep} - \varepsilon_{initial}$ and contours of the streamfunction (streamlines). In the following figures, contours of ε_{pert} are placed at the top, and the streamline plots underneath in a mirror image orientation. In this way the change in vorticity, which moves through the cylinder in conjunction with the change in swirl, can be directly related to the onset of vortex breakdown.

Results were generated using the commercial CFD package Fluent version 4.4.8. The pipe geometry was generated using Geomesh, part of the Fluent package. The pipe wall was specified as slip, the axis as a symmetry axis. The inlet boundary condition was assigned the axial and azimuthal velocity profiles given in equation (1), and the outlet was specified as an outlet boundary, and as such is restricted to outflow only. Fluent 4.5 uses a finite volume formulation, and a QUICK differencing scheme was used. For the time dependent part of the study, a 1st order implicit method was used. The timestep was nondimensionalised based on the inlet vortex core radius, r_c=1, and the inlet freesteam axial velocity, U_f=1. .A timestep of 0.025 was found to be appropriate – smaller timesteps down to 0.01 gave quantitatively similar results to within a few percent of the results presented, where the value of the minimum streamfunction value at each timestep was used to compare solutions.

An open pipe with a non-uniform profile was used, and consisted of a converging then diverging inlet, a straight test section, and a converging outlet. Axisymmetry was assumed, since only the axisymmetric bubble form of breakdown is observed at the Reynolds number and swirls considered. Figure 2 shows the geometry and grid (the grid in figure 2 is much coarser than the actual grid used, which was 296x38 cells):

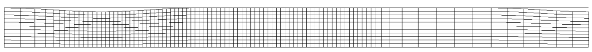

Figure 2: Pipe geometry and grid.

The mesh has been compressed in the axial direction in the region where breakdown is expected in order to capture the large velocity gradients there. An earlier study confirmed the accuracy of the model considered here – comparison of bubble size and location with results obtained by Beran and Culick (1992) show good agreement in both respects.

RESULTS

Figure 3 shows streamlines for two cases in the hysteresis region.

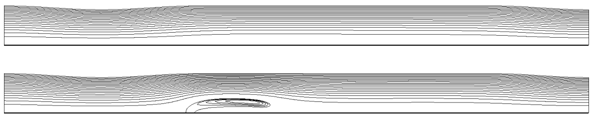

Figure 3: Solutions for Ω=1.45

The first has Ω=1.45, and no breakdown bubble - this solution was reached by increasing the swirl from Ω=1.34. The second figure is also for Ω=1.45, but in this case a bubble is present - this solution was reached by reducing the swirl from a breakdown flow at Ω=1.461. Ω=1.45 at Re=1000 is in the body of the hysteresis region (see figure (1)).

Figure 4 shows the evolution of the flow following the increase and subsequent decrease in swirl. As mentioned previously, the top plot shows the azimuthal vorticity - this quantity is plotted in 15 equally spaced contours. The contour levels assigned to each plot are the same. The streamfunction contours below are not equally spaced - there are 5 contours of negative streamfunction values, and 20 contours of positive streamfunction values. This unequal partition is assigned to ensure the breakdown bubble is visible when it evolves. Again, the contour levels assigned to each plot are the same. Figure 4(a) and (b) show the increase in swirl entering the pipe at the inlet. By figure 4(c) the inlet swirl has reverted to the initial condition, and the swirl change is propagating downstream.

Although only an increase in Ω is introduced at the inlet, resulting in an increase in the axial component of vorticity, there is an immediate increase in the azimuthal component as well. This azimuthal component may be the result of turning of axial vorticity into the azimuthal direction - investigation of the generation term is required in order to determine the mechanism operating here, along the lines of Jones *et al.* (1999) for a confined cylinder flow.

As the change in azimuthal vorticity propagates through the pipe constriction, its magnitude increases. This is expected - since the circulation around a vortex tube is conserved, the change in radius of the vortex tube must result in a change in the vorticity magnitude.

After the decrease in Ω to the initial condition level, the entire change begins to move through the converging section. At the diverging section, some of the change continues to propagate downstream, while a large portion slows down near the location where breakdown subsequently evolves. This is the behaviour observed by Darmofal and Murman, where a portion of the azimuthal vorticity moves through the pipe, while some remains trapped at a certain axial location. In their study, this trapped portion grows in amplitude and eventually leads to the formation of a breakdown bubble (figure 4(e)). From the initial change in vorticity (figure 4(a)), streamline divergence is observed in association with the change in azimuthal vorticity, signifying a region of retarded axial flow. This region moves downstream with the front of the vorticity change. In figure 4(d) the streamline divergence takes on a sharper aspect - this occurs just upstream of the axial location where breakdown subsequently develops.

The flow continues to develop, until at figure 4(e) a breakdown bubble forms. With increasing time the bubble moves downstream with the azimuthal vorticity change, and decreases in size and strength as it moves further away from the pipe constriction. Eventually, at figure 4(g) the bubble completely disappears from the flow, although a small streamline divergence persists.

After the final result shown here, the azimuthal vorticity change propagates through to the end of the pipe and out the outlet. A portion of the change remains trapped for a short time around the converging-diverging section, but then moves downstream and exits the pipe.

Although the bubble seems to represent a minimum in the azimuthal vorticity, the greatest reduction in azimuthal vorticity relative to the initial condition does not correspond to the breakdown bubble. The minimum occurs at the same axial location as breakdown, but is a greater radial distance away from the axis than the bubble.

DISCUSSION

In Darmofal and Murman's (1994) study, the increase in Ω was maintained, while here the increase is kept on for 10 timesteps, after which the inlet boundary condition reverts to its initial state. We observe the slowing of some of the azimuthal vorticity at the location where breakdown subsequently develops, and the evolution of a breakdown bubble, but instead of growing to a steady state, the bubble disappears, and all of the azimuthal vorticity perturbation propagates downstream and out of the pipe.

The primary aim of this study was to investigate the hysteresis inherent in the open pipe at the Reynolds number and swirl numbers considered. The intention was to determine whether it was possible to make the flow jump from an initial state where no breakdown was observed to a conjugate state at the same Reynolds number and swirl where a bubble exists. The evolution of a breakdown bubble in a swirl increase was expected - an increase in swirl would result in a bubble in any case, as long as the increase took the flow outside the hysteresis region. The question to be answered was whether the flow would then revert back to the initial condition once breakdown evolved, or stay in its new state and retain the breakdown bubble. The results presented here show that a bubble is generated with a transient increase in swirl, but once the swirl increase has passed the location where a bubble can be maintained, the bubble disappears.

These results suggest that while the flow with breakdown is relatively stable if approached from a higher swirl result generated outside the hysteresis region, the flow with breakdown, if it is generated by perturbing another solution in the hysteresis region, is not very robust. Further work will concentrate on performing the same jump in swirl at a lower Reynolds number, as at lower Reynolds numbers the difference between the no-breakdown and breakdown solutions is much smaller. In fact, at small enough Reynolds number the hysteresis is not present.

In examining the behaviour of the azimuthal vorticity in relation to the onset of vortex breakdown, it is important to consider the origin of this component of vorticity in the open pipe. Initially, as the increase in swirl is applied to the inlet, the only vorticity components which are directly affected are the axial and radial components. However, it was observed here that a change in the azimuthal vorticity is evident at the pipe inlet – this change can only be a product of some transformation of the axial and radial vorticity components. Darmofal (1993) examined vorticity dynamics in the open pipe, and showed that tilting of the axial component of vorticity was the major source of azimuthal vorticity generation. The axial vorticity tilting is caused by an axial pressure gradient. Retardation of the axial flow caused by the pressure gradient results in streamline divergence, due to continuity. This streamline divergence forces tilting of the axial vorticity vector, which results in the generation of azimuthal vorticity. The azimuthal vorticity generated serves to increase axial flow retardation, resulting in more streamline divergence, more axial vorticity tilting, and more azimuthal vorticity generation. The system feeds back until a stagnation point appears on the pipe centreline, and a recirculation region develops along the axis – the breakdown bubble.

CONCLUSION

Previous studies (Beran and Culick, 1992) have identified the hysteresis present in the open pipe for a range of Reynolds numbers and swirl ratios. The work presented here has explored this region of parameter space for the open pipe, and attempted to push the flow from one state, which does not display vortex breakdown, to its conjugate state, in which a breakdown bubble is evident. This preliminary investigation has showed that while it is possible to push the flow to breakdown by introducing a large transient change in the swirl, the resulting bubble does not develop to the same extent as the bubble in the steady flow. Also, once the transient perturbation has passed through the region where a breakdown bubble generally forms, the bubble disappears. The wave trapping observed by Darmofal and Murman (1994) has been qualitatively verified here, although the azimuthal vorticity change does not remain in the flow, but eventually propagates to the outlet.

Further work will concentrate on the hysteresis manifest at lower Reynolds numbers, since at lower Reynolds number the difference between the no breakdown and breakdown flows is not so large, and hence it may be easier to trip the flow into the other hysteresis state. The transition from breakdown to no breakdown will also be considered, to determine whether there is symmetry between the two transitions. It will be investigated whether transient perturbations can lead to vortex breakdown suppression.

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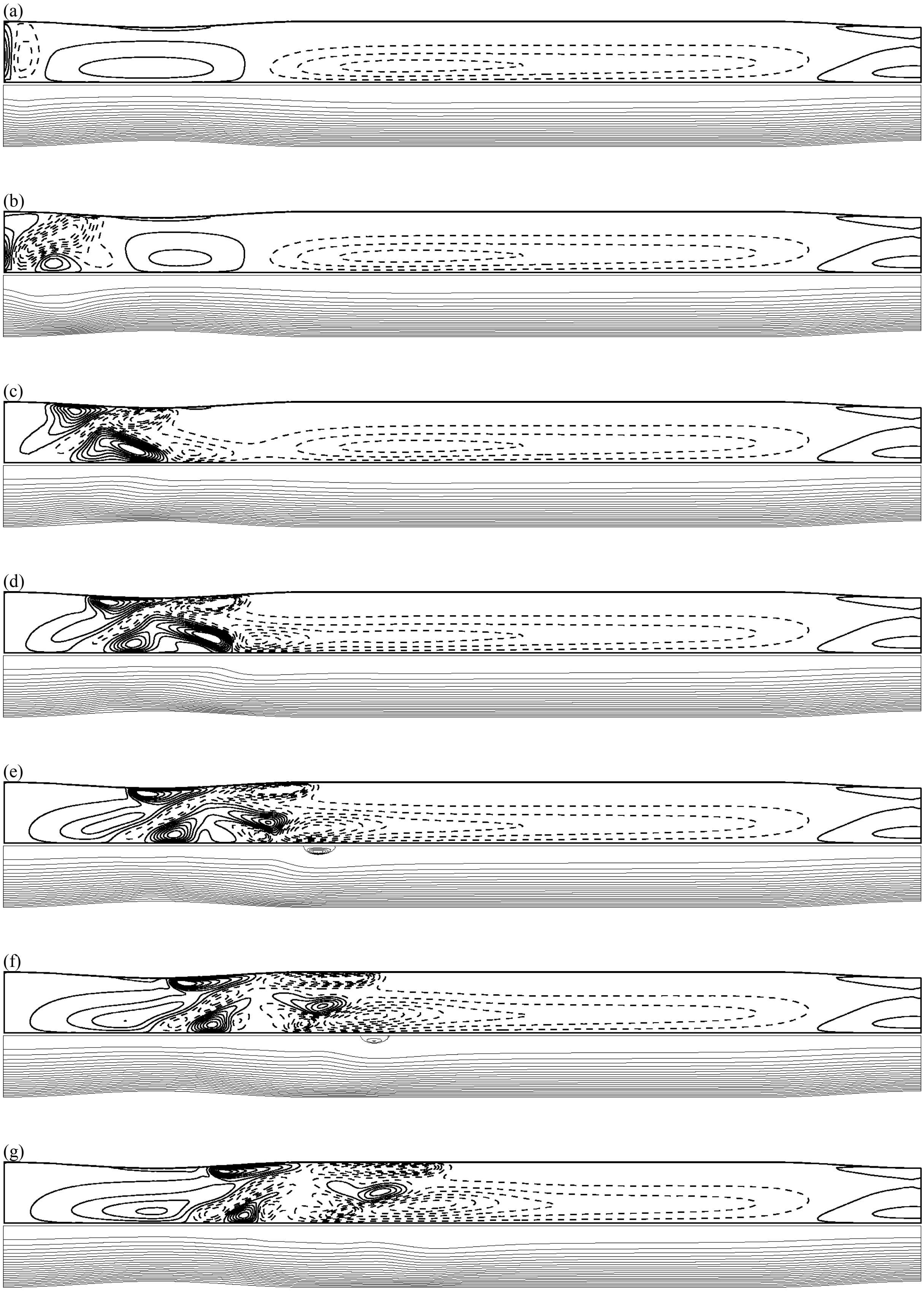

Figure 4: Azimuthal vorticity change (top) and streamfunction contours (bottom) for timesteps (a) 20, (b) 80, (c) 140, (d) 200, (e) 260, (f) 320, (g) 380